British Journal of Mathematical and Statistical Psychology (2010), 63, 575–601

The British Psychological Society

www.bpsjournals.co.uk

Coping with stress through decisional control: Quantification of negotiating the environment

Matthew J. Shanahan* and Richard W. J. Neufeld
The University of Western Ontario, London, Canada

Coping with stress through 'decisional control' – positioning oneself in a multifaceted stressing situation so as to minimize the likelihood of an untoward event – is modelled within a tree-structure scenario, whose architecture hierarchically nests elements of varying threat. Analytic and simulation platforms quantify the game-like interplay of cognitive demands and threat reduction. When elements of uncertainty enter the theoretical structure, specifically at more subordinate levels of the hierarchy, the mathematical expectation of threat is particularly exacerbated. As quantified in this model, the exercise of decisional control is demonstrably related to reduction in expected threat (the minimum correlation across comprehensive parameter settings being .55). Disclosure of otherwise intractable stress-coping subtleties, endowed by the quantitative translation of verbal premises, is underscored. Formalization of decisional stress control is seen to usher in linkages to augmenting formal developments from fields of cognitive science, preference and choice modelling, and nonlinear dynamical systems theory. Model-prescribed empirical consequences are stipulated.

1. Introduction

Human dealings with psychological stress are pervaded by cognition-intensive coping (Holahan, Holahan, Moos, Brennan, & Schutte, 2005; Wilson, MacLeod, Mathews, & Rutherford, 2006). A prominent and ubiquitous form of such coping is 'decisional control' (Averill, 1973; Thompson, 1981). Decisional control entails positioning oneself in a multifaceted stressing situation so as to minimize the probability of an untoward event, such as the occurrence of physical danger or discomfort, or an abrasive social interchange (Lees & Neufeld, 1999). Implicit in decisional control is increased accessibility to less threatening options, should an effective decision be made. As the number of options increases, the likelihood of being barred from the one carrying minimal threat decreases (assuming no bias in availability; Neufeld & Paterson, 1989). Increased, however, is the number of predictive judgements required to ascertain the

* Correspondence should be addressed to Matthew J. Shanahan, Department of Psychology, The University of Western Ontario, Westminster Hall, London, Ontario, Canada N6A 3K7 (e-mail: mshanah@uwo.ca).

DOI:10.1348/000711009X480640

least-threat option from amongst those available (Neufeld, 1982). Such 'cognitive work' incurred by decisional control is held to be stressing in its own right (e.g., Solomon, Holmes, & McCaul, 1980; Wright, 1984). This apparent structure of decisional control invites a choice-based game-like analysis of the interplay of decisional control's threat-reducing and cognitive-load properties, an analysis deemed important for the following reasons.

Decisional control theoretically brings to bear 'exogenous and endogenous sources of stress': untoward-event threat and information-processing demands. Evaluating the tenability of reciprocality for these sources – reduction in one potentially being accompanied by increase in the other – requires a quantitative representation of each. Alternately, empirical expression of these respective agents of stress may be registered on separate stress indices (Kukde & Neufeld, 1994; Tomaka, Blascovich, Kibler, & Ernst, 1997). Ascertaining the requisite measure-theoretic sensitivity and specificity of dissociable empirical indices requires a 'gold standard', again comprising quantification of their associated, exogenous versus endogenous, sources of variation.

Formal delineation of decisional-control properties also indicates the composition of threat-predictive judgements requisite to threat-minimizing choices, specifically cognitive processes (e.g., visual search concerning choice alternatives, and memory search related to their threat properties) underlying predictive efficacy. A by-product of such analysis comprises potentially important insight regarding sources of compromised stress negotiation associated with selected cognitive deficits in psychopathology (Neufeld, Boksman, Vollick, George, & Carter, 2010).

The operation of non-organismic barriers to choice-based threat reduction also stands to be unveiled with formal implementation of decisional-control structures. Such barriers entail outcome uncertainty, where environmental exigencies are such as to prevent pre-knowledge regarding specific consequences of available choice (Paterson & Neufeld, 1995). The nature of such potential 'structural threats to threat reduction' in principle can be dissected through their translation into quantitative prototypes.

Its formalization also translates decisional control into a format conducive to estimation of individual predilection to its corresponding cognitive demands and threat-reducing returns. Certain formal models of preference and choice, arguably lending themselves to the structure of decisional control – e.g., Batsell, Polking, Cramer, and Miller, (2003); Tversky's (1972a, 1972b) 'elimination by aspects' (EBA); cf. Saari (2005) – require an informed designation of judged items' attributes. As instantiated here, situations varying in decisional-control attributes constitute the objects of judgement, while the attributes themselves tenably consist of potential threat reduction, and of cognitive transactions essential to its realization (Morrison, Neufeld, & Lefebvre, 1988). Rigorous expression of individual differences in amenability to decisional control in turn stands to catalyse linkages to candidate psychometric measures, and their associated constructs – e.g., 'desirability for control' (Burger & Cooper, 1979); 'need for cognition' (Cacioppo, Petty, & Kao, 1984); 'coping with uncertainty' (Greco & Roger, 2001); and the 'Big Five' personality traits (Connor-Smith & Flachsbart, 2007).

Other formal approaches to stress, coping, and related variables have invoked nonlinear dynamical systems theory (NLT) and its associated computational modelling methodology (e.g., Booth, 1985; Guastello, 1992; Levy *et al.*, 2010; Neufeld, 1999a; see also Yao, Yu, Essex, & Davison, 2006, pp. 506–508). Here, dynamical differential equations give rigorous mathematical expression to long-held verbal conjectures about temporal interactions comprising 'stress as a dynamical process' (Monroe, 2008; enumerated in Neufeld, 1999a). Notably, in contraposition to the present analysis, the informational yield of NLT modelling is not contingent on stipulation of the precise mechanisms whereby system variables (e.g., level of stressors amenable to decisional control; level of decisional-control engagement) impact on each other. Although NLT modelling can be shown to have all the ingredients beckoned by advocates of a 'stress-as-a-process' theoretical orientation – ingredients recently restated by Monroe (2008) – complementing avenues of modelling, including the subject of this paper, can speak more intricately to the mechanisms whereby the depicted dynamical interplay may occur.

In the search for an analytic platform quantitatively linking threat reduction and cognitive load and providing a normative model for their interplay, we present the model below and instantiate it across a broad spectrum of parameter values. Both beneficial decisional structure attributes and their opposites, the 'situational threats to threat reduction', are clearly delineated as quantifiable aids or impediments to a decision maker's (DM's) threat-reducing efficacy.

2. A model of decisional control

2.1. Exemplified model structure and assumptions

A brief example from a mental health setting, where stress and coping considerations abound, will outline the model structure and key assumptions (see online Appendix, http://publish.uwo.ca/~mshanah for supplementary technical details). Consider an authorized DM evaluating long-term care options for a mental health patient. For simplicity, the only consideration for the DM is minimizing the threat of relapse (a *maximizing* strategy (for likelihood of long-term stability); Janis & Mann, 1977; Rappaport, 1983). Increasing threat values t_i for relapse are known, unique, and discriminable among q floor units ('elements'), nested within p care facilities ('bins'):

$$i = 1, 2, \ldots, pq;$$

$$\{t_1 < t_2 < \ldots < t_i < \ldots < t_{pq}\}$$

$$t_j > t_i \text{ iff } j > i.$$

The model accords the DM varying choice conditions at each node (floors or facilities). The notation C corresponds to unfettered *choice*; U, *uncertainty*, corresponds to external assignment (i.e., by a health organization or government body) of a selection at a given node, the DM's knowledge of which is *deferred* until after all decision making has taken place; N, for *no choice*, refers to external assignment, the knowledge of which is *available* for the DM's decision making at other nodes. When external assignment is made (U or N) at a given node, each option among q floors or p facilities has an equal chance of being assigned. Each facility is assumed to have the same number of floors (and similarly if facilities are further nested within P towns – 'bin sets' – for example). Advantageous tractability results from these latest two assumptions, while nevertheless preserving generality. The model assumes that a selection must be made among available options, and retreat to an alternate selection is impossible once deferred external assignments (U) are revealed. Finally, 'information-processing demand' is estimated on the assumption that, due to the importance of the threat, anticipation (evaluation) of potential encounters (i.e., floor assignments) remains operative, even when choice cannot be exercised (condition U). To describe a scenario, choice conditions are ordered from higher to lower nesting. Thus, free choice of facility with deferred assignment of floor would be denoted CU (Morrison *et al.*, 1988; Neufeld, 1982; see also Averill, 1979; Thompson, 1981).

2.2. Delineation of decisional-control attributes

Decisional control/threat reducibility is defined as the probability of access to the situation's least threatening option, $\Pr(t_1)$; information-processing demand as potential outcome set size (OSS); and degree of objective threat as the mathematical expectation of threat, $E(t)$, attending the exercise of decisional control under prevailing choice constraints. (*Note*. A related decisional situation attribute, unpredictability of threatened events, $\mathrm{Var}(m)$, is formally defined and discussed in the online Appendix, http://publish.uwo.ca/~mshanah).

2.2.1. Decisional control and threat reducibility

Given selection-outcome contingency, more pre-emptive control can be exerted with more available choices (Paterson & Neufeld, 1995). Available decisional control is defined here as the number of possible responses the DM can make – response set size (RSS). Under a maximizing strategy, RSS correlates perfectly with the probability of access to the least threatening option, $\Pr(t_1)$. The relationship between the two quantities is expressed for a first- or second-order scenario with p bins and q elements, or P bin sets, p bins per set, and q elements:

$$\mathrm{RSS} = pq\cdot\Pr(t_1) \quad \text{or} \quad \mathrm{RSS} = Ppq\cdot\Pr(t_1).$$

2.2.2. Information-processing demands

Information-processing demands are defined as a potential OSS. For p bins and q elements in each bin, for example, there are pq possible outcomes under conditions CC, CU, UC, and UU; q under NC and NU; p under CN and UN, and 1 for NN. The assumed maximizing strategy centres on threats with non-negligible aversiveness, hence exhaustive processing of potential encounters (OSS) is assumed, even when RSS $<$ OSS. In condition CU, for instance, the bin with the lowest t_i value (in this case t_1) would be ascertained and chosen.

As a measure of information-processing demands, OSS allows for comparison with threat reducibility, potentially revealing implications for individual differences in decisional-control predilection and amenability (Morrison *et al.*, 1988).

2.2.3. Mathematical expectation of threat

$E(t)$ conveys the amount of threat prevailing in the aftermath of exploiting decisional control as availed by the governing choice conditions. It incorporates the product of the probability of access to the least threatening option and the value for this lowest threat level, with the products of all other probabilities and corresponding threat levels, altogether yielding threat expectation (expected probability of an untoward event).

2.3. Sample derivation of quantities

A narrative of the derivation of the quantities for a first-order scenario is provided below. Quantities for RSS, $\Pr(t_1)$, and OSS for a representative sample of 10 scenarios are presented in Table 1; corresponding expressions for $E(t)$ are presented in Table 2. Note that these 10 expressions comprehensively depict different general forms of equations for $E(t)$ among first- and second-order scenarios.

Table 1. Derived quantities for 10 representative scenarios

Scenario architecture	RSS	$\Pr(t_1)$	OSS
CC	pq	1.0	pq
NN	1	$1/(pq)$	1
CU	p	$1/q$	pq
NC	q	$1/p$	q
CCC	Ppq	1.0	Ppq
NNN	1	$1/(Ppq)$	1
CCU	Pp	$1/q$	Ppq
CCN	Pp	$1/q$	Pp
CNU	P	$1/(pq)$	Pq
UCN	p	$1/(Pq)$	Pp

Note. C, free choice over constituents of the associated tier; U, constituent assignment, with uncertainty as to assignment, pending tier entry; N, constituent assignment, with foreknowledge of assigned constituent.

2.3.1. Scenario NC

$$\Pr(t_1) = 1/p;$$

$$\text{RSS} = q;$$

$$E(t) \text{ is stated in Table 2;}$$

$$\text{OSS} = q.$$

For NC, $\Pr(t_1)$ reflects the probability of external assignment (no choice – N) of the bin containing t_1. The RSS reflects the number of elements available for selection under the free-choice (C) condition. The expected threat derivation involves the use of the hypergeometric distribution (Patil & Joshi, 1968; specifics of current implementation are available in the online Appendix, http://publish.uwo.ca/~mshanah, together with a narrative for a sample second-order scenario, UCN). There are q elements in the assigned bin, hence the value of OSS.

3. Simulation methods

For the first-order scenarios, nine bin-element scenario structures were available as the cross product of the three choice conditions, C, U, and N, at the element-nesting bin level of the structure, with C, U, and N at the nested-element level. The number of Cartesian product elements entailing choice conditions rose to 27 for the second-order scenarios. The mathematical computing platform MATLAB (version 7.2.0.232 [R2006a]) was used to calculate values of decisional-control scenario attributes and their associations. The input for an individual scenario structure comprised a value for a p and q pair, or a P, p, and q triplet, a value for t_1 and for the increment between t_i values.

Table 2. Expected threat E(t) for 10 representative scenarios

Scenario architecture	Formula for E(t)
CC	t_1
NN	$\frac{1}{pq}t_1 + \frac{pq-1}{pq}\cdot\bar{t}_{i'}$ [a]
CU	$\frac{1}{q}t_1 + \frac{q-1}{q}\cdot\bar{t}_{i'}$
NC	$\frac{1}{p}t_1 + \frac{p-1}{p}\cdot\frac{q}{pq-1}\sum_{i'=2}^{pq} H(q-1;pq-2,pq-i',q-1)\cdot t_{i'}$
CCC	t_1
NNN	$\frac{1}{Ppq}t_1 + \frac{Ppq-1}{Ppq}\cdot\bar{t}_{i'}$
CCU	$\frac{1}{q}t_1 + \frac{q-1}{q}\cdot\bar{t}_{i'}$
CCN	$\frac{1}{q}t_1 + \frac{q-1}{q}\cdot\left(\frac{q-1}{Ppq-1}\cdot\frac{1}{q-1} + \frac{(Pp-1)q}{Ppq-1}\cdot\frac{1}{q}\right)\sum_{i'=2}^{Ppq} H(Pp-1;Ppq-2,Ppq-i',Pp-1)\cdot t_{i'}$ [b]
CNU	$\frac{1}{P}\left(\frac{1}{q}t_1 + \frac{q-1}{q}\cdot\bar{t}_{i'}\right) + \frac{P-1}{P}\cdot\frac{Pq}{Ppq-1}\sum_{i'=2}^{Ppq}\left\{H(Pq-1;Ppq-2,Ppq-i',Pq-1)\cdot\frac{1}{q} + (1-H(Pq-1;Ppq-2,Ppq-i',Pq-1))\frac{q-1}{Pq-1}\cdot\frac{1}{q}\right\}t_{i'}$
UCN	$\frac{1}{P}\left(\frac{1}{q}t_1 + \frac{q-1}{q}\cdot\frac{p}{Ppq-1}\sum_{i'=2}^{Ppq} H(p-1;Ppq-2,Ppq-i',p-1)\cdot t_{i'}\right) + \frac{P-1}{P}\cdot\frac{(P-1)p}{Ppq-1}\cdot\frac{1}{P-1}\sum_{i'=2}^{Ppq} H(p-1;Ppq-2,Ppq-i',p-1)\cdot t_{i'}$

[a] The term $\bar{t}_{i'}$ denotes the mean of $pq-1$ values of $t_{i'}$, in the composition of E(t) for NN and CU, and the mean of $Ppq-1$ values of $t_{i'}$, in the composition of E(t) for NNN, CCU, and CNU; see the note to Table 1 for definitions of C, U, and N; P, number of bin sets; p, number of bins nested in bin sets; q, number of elements nested in bins.

[b] The expression in braces reduces to $Pp/(Ppq-1)$. As in the text, expansion conveys a potentially informative 'interplay' among constituent terms: the probability of $t_{i'}$ being in the bin containing t_1, $(q-1)/(Ppq-1)$, is multiplied by the probability of $t_{i'}$'s assignment, given both its location in t_1's bin and non-assignment of t_1, $1/(q-1)$, plus the probability of being in a bin other than t_1's bin, given non-assignment of t_1, $(Pp-1)q/(Ppq-1)$, multiplied by the probability of $t_{i'}$ being assigned, given its location outside t_1's bin, $1/q$. Similar considerations apply to other listed E(t) formulae.

This led to a pq- or Ppq-tuple vector of t_i values, and computation of the scenario's attribute quantities of Pr(t_1), E(t), and OSS. Two extensive lists exhausted possible arrangements of p and q, and P, p and q values such that, respectively:

$$pq \le 100; \quad p,q \ge 2; \quad p,q \in N,$$
$$Ppq \le 100; \quad P,p,q \ge 2; \quad P,p,q \in N.$$

These constraints generated 283 pairs of p and q values and 324 triplets of P, p, and q values. The number of runs totalled 2,830 for the set of nine first-order scenario structures, and 1,944 for the set of 27 second-order scenario structures.[1]

[1] Complete code for these simulations is available from the first author.

4. Specific analyses

4.1. **Pr(t_1)** *as an index of mathematical expectation of threat,* **E(t)**

There is computational and empirical evidence that the probability of access to the least threatening option Pr(t_1) can serve as an approximation to the mathematical expectation of threat E(t) (Kukde & Neufeld, 1994; Morrison *et al.*, 1988). Pr(t_1) is calculated as the inverse of the product of the number of items at all tiers with constraint on choice. In the case of CCU, for example, Pr(t_1) is simply $1/q$, q being the number of elements available in the U condition. Pr(t_1) is thus fairly straightforward to calculate, and is inversely proportional to q.

The calculation of E(t) for CCU is more involved and less readily apparent:

$$\mathrm{E}(t) = \frac{1}{q}t_1 + \frac{q-1}{q}\cdot \bar{t}_{i'} \tag{1}$$

(see note 'a' of Table 2).

For other scenario architectures (e.g., UCN, CNU), a computing aid is unequivocally required to evaluate E(t), especially to implement the hypergeometric distribution. However, Pr(t_1) remains straightforward to calculate, as the inverse of at most the product of two (first-order structure) or three (second-order structure) natural numbers. Finally, Pr(t_1) is a scaled value of RSS, the index of available decisional control. Decisional control, also termed threat reducibility, ought by definition to impact the mathematical expectation of threat. Our first analysis was designed to quantify this association, and was directed towards examining Pr(t_1) as a tractable, tenable surrogate for E(t).

4.1.1. Results

Equivalence of the correlations across all t_1 and Δt_i values accorded with the 'scalar effect' of level of threat, such that it had no influence on correlations within specific values of the scenario-structure parameters p, q, or P, p, q. The result is consequential to each set of t_i values forming an intended discrete uniform distribution, whereby the contour of structurewise E(t)s remains stable. (A non-trivial by-product of this observation is its attestation to computational accuracy of the simulations.)

The average correlation between Pr(t_1) and E(t) for the first-order scenarios was $-.7698$, with a standard deviation (computed across combinations of scenario-structure parameter values) of .1045. Values ranged from $-.5894$ ($p = 11$, $q = 9$), to $-.9527$ ($p = q = 2$). Average correlation for the second-order scenarios was $-.6851$, the standard deviation being .0753. The weakest correlation was $-.55$ ($P = p = 5$, $q = 4$), the strongest being $-.8876$ ($P = p = q = 2$).

The comprehensive examination of the Pr(t_1) to E(t) correlation supports, if imperfectly, the negative correspondence of Pr(t_1) with the benchmark threat value E(t). Even the weakest specific correlation, $r = -.5500$, indicates that Pr(t_1) accounts for 30% of the variance in E(t). If the descriptors of central tendency are used, the average correlations indicate that Pr(t_1) in general accounts for 59% and 47% of the variance in E(t) in first- and second-order scenarios, respectively. The difference between the weakest and strongest value suggests the association varies with regions of the scenario-structure parameters. In the present case, the strongest associations accompanied the lowest values for p, q, and P, p, and q; the weakest associations accompanied parameter values equal to or approaching the current upper limit of their product, with approximate evenness of these values across tiers of the hierarchical scenario structure.

To aid in interpreting the simulation analysis, a brief illustration will be given in the context of the hypothetical mental health example presented earlier. In the case of $\Pr(t_1)$, as a negative indicator of $E(t)$, the implication is that by and large it is desirable to maximize $\Pr(t_1)$ by reducing the number of potential encounters at decision nodes without free choice, because $\Pr(t_1)$ is the inverse of the product of the potential encounter set size at nodes with external assignment. Put simply from the individual's perspective, one is likely to have more impact on reducing threat if there are fewer items that could be externally assigned at each hierarchy level. There must of course be free choice (C) at a minimum of one node in the scenario for decisional control to be available at all. As an example, when choosing among several facilities each with several in-patient floor units within a CN decision structure, the ability to select a facility to minimize likelihood of relapse would be enhanced if the number of floors per facility (or q elements at the node with N (no choice)) were minimized, thus maximizing $\Pr(t_1)$.

4.2. Associations of information-processing demands (OSS) with threat reducibility $\Pr(t_1)$ *and expected threat,* $E(t)$

Information-processing demands are theorized to play a role in stress negotiation, especially under the present maximizing strategy, where options are exhaustively evaluated. From earlier sample calculations (Neufeld, 1999b), a moderate correlation between $\Pr(t_1)$ and OSS is expected as reflecting some association between threat reducibility and number of potential outcomes; a low moderate negative correlation between $E(t)$ and OSS is predicted. As such, expected threat would be projected to increase somewhat with a decrease number of potential outcomes.

4.2.1. Results

The average correlation between $\Pr(t_1)$ and OSS for the first-order scenarios was .4452, with a standard deviation of .0168. Values ranged from .4267 ($p = 10$, $q = 10$), to .4875 ($p = q = 2$). The average correlation for the second-order scenarios was .4359, the standard deviation being .0139. The weakest correlation was .4082 ($P = p = 5$, $q = 4$), the strongest being .4751 ($P = p = q = 2$).

Values for OSS evidently are moderately correlated with $\Pr(t_1)$, within a relatively narrow range, indicating a robust moderate correlation. The maximum and minimum associations are identified with patterns of parameter values closely related to those for $\Pr(t_1)$ and $E(t)$, above.

For the first-order scenarios, the average correlation between OSS and $E(t)$ was $-.2915$, with a standard deviation of .0661. Values ranged from $-.1873$ ($p = q = 10$) to $-.4173$ ($p = q = 2$). The average for the second-order scenarios was $-.2072$, with a standard deviation of .0426. Values here ranged from $-.1347$ ($P = p = 5$, $q = 4$) to $-.3305$ ($P = p = q = 2$).

The association of OSS and $E(t)$ in turn is decidedly present, albeit the weakest of the three inter-attribute associations. The range of associations is greater than that for $\Pr(t_1)$ and OSS; maximum and minimum associations again follow the configuration of parameter values attending maximum and minimum associations for the other pairs of attributes.

In terms of the mental health care example, it can simply be stated that a multiplicity of potential encounters (OSS) has some, though generally weaker, relation to reducing expected threat $E(t)$ through decisional control. Specifically, a large total set of potential floor units on which the patient might be placed does not in itself confer much decisional-control advantage. The quantity OSS is calculated from the product of the set size for nodes in the C or U condition. Advantage accrues from maximizing the set size for nodes with free choice (C), as these also confer decisional control, and minimizing the set size for nodes with uncertainty (U), as these do not. For our mental health treatment example, assuming for illustration a 'CUN' scenario of P towns (C condition) nesting p facilities (U condition) nesting q floor units (N condition), optimizing the OSS components' effect on E(t) lies in maximizing P (towns) and minimizing p (facilities) per town.

4.3. Uncertainty and threat

Within this model architecture, uncertainty instigates certain processing exigencies amidst nevertheless curtailed threat reducibility. With uncertainty, there are increased potential outcomes to evaluate (relative to N), yet reduced information upon which to base a decision (again relative to N). Additionally, relative to C, there is hampered freedom of action to implement optimal choice. Thus, an examination of scenarios ranked from top to bottom in order of progressively increasing E(t), with a focus on condition U, speaks to uncertainty as compromising stress negotiation. Findings from these analyses are summarized in Tables 3 and 4, for the first- and second-order scenarios, respectively.

4.3.1. Results and implications

The most salient adverse effect of uncertainty U on E(t) occurs when C at a superordinate level is followed by U at a subordinate level of the scenario structure. Simulations containing a CU ordering invariably have higher E(t) values than those with the equivalent number of conditions C, U, N, but with reverse ordering of C and U in the structure's hierarchy. Thus in the illustrative results of E(t) reported in Tables 3 and 4, where the broad range from $t_1 = .1$ to max $t_i = .99$ affords considerable scope for reducing stress by the exercise of decisional control, the advantage of any other arrangement involving at least one free choice (i.e., one C) over CU orderings is apparent. For example, UC generates an E(t) of .2242, as compared to CU's E(t) of .4625.

An informative comparison can be made for scenarios framed identically, except that N replaces U. In many of these comparisons, there is little or no effect of substituting N for U. For example, UCC and NCC have identical average E(t) values. A similar situation exists for UUC and NNC. However, considerable difference in average E(t) emerges where N is substituted for U at a level subordinate to a C condition, especially in the most subordinate level. This is most evident in comparing CCN (E(t) = .1519) and CCU (E(t) = .4151). Moreover, U positioned subordinate to C can undermine an increased collective presence of C, as seen in E(t) for CCU, above, as opposed to CNN (.2887).[2] An illustration of the impact of the positioning of the uncertainty condition is provided in Section 5.4 in the form of an organizational example, specifically related to Karasek's (1979) job demand-control model.

Scenario structures clustering together in Tables 3 and 4 can be examined for corresponding coalescence of their E(t) structures. Doing so is facilitated with an

[2] All patterns of results remained stable with simulation constraints comprising $q \geq p(\geq P)$; such ordering may more closely parallel naturally occurring hierarchy patterns.

Table 3. List of first-order scenario architectures by increasing mean E(t)

First-order scenarios equivalent in mean E(t)	Mean E(t) for selected t range over exhaustive p, q list[a]	Maximum E(t) for any one set of p, q values	p, q values for maximum E(t)	Minimum E(t) for any one set of p, q values	p, q values for minimum E(t)
CC[b]	.1000	.1000	all values	.1000	all values
CN, UC, NC	.2242	.3937	2,50;50,2;50,2	.1088	50,2;2,50;2,50
CU	.4625	.5405	2,50	.3247	21,2
UU, UN, NU, NN[c]	.5450	.5450	all values	.5450	all values

Note. See notes to Tables 1 and 2 for definitions of C, U, N, p and q.

[a] For $t_1 = .1$, $\max t_i = .99$, exhaustive p, q values (all 283 possible pairs within specified constraints), and $\Delta t_i = (\max t_i - t_1)/(pq - 1)$.

[b] Identical row values result from free access to $t_1 = .1000$.

[c] Identical row values result from the t_i forming a discrete uniform distribution, with mean $=$ range$/2 + t_1$, regardless of structure-parameter values.

Table 4. List of second-order scenario architectures by increasing mean E(t)

Second-order scenarios equivalent in mean E(t)	Mean E(t) for selected t range over exhaustive P, p, q list[a]	Maximum E(t)	Minimum E(t)
CCC[b]	.1000	.1000	.1000
CCN, CNC, UCC, NCC	.1519	.2726	.1088
CUC	.2454	.3877	.1173
CNN, UCN, UUC, UNC, NCN, NUC, NNC	.2887	.3937	.1259
CCU	.4151	.5315	.3247
CUN, CNU, UCU, NCU	.4507	.5361	.3314
CUU	.5155	.5405	.4371
UUU, UUN, UNU, UNN, NUU, NUN, NNU, NNN[c]	.5450	.5450	.5450

Note. P, p, q coordinates are not given for maximum and minimum E(t) as the coordinates are not singular within scenario structures; see notes to Tables 1 and 2 for definitions of C, U, N, P, p, and q.
[a] For $t_1 = .1$, $\max t_i = .99$, exhaustive P, p, q values (all 324 possible triplets within specified constraints), and $\Delta t_i = (\max t_i - t_1)/(Ppq - 1)$.
[b] Identical row values result from free access to $t_1 = .1000$.
[c] Identical row values result from the t_i forming a discrete uniform distribution, with mean = range/2 + t_1, regardless of structure-parameter values.

available summarizing syntax implementing the current computational structures of E(t) (Morrison *et al.*, 1988; Neufeld, 1999b). It is potentially instructive to note transitions in these summary expressions across the scenario clusters. A clusterwise listing is presented in Appendix B.

4.3.2. Dissection of uncertainty effects

Simulation results evincing adverse effects of choice condition U on threat reducibility, saliently exacerbated by U's positioning at lower levels of the scenario structure's nesting hierarchy, invite inspection of related formulae. The quantitative workings of the presence of condition U's 'information concealment' are instantiated in structure CUC, over and against CNC; those of U's lower-tier positioning, in turn, are instantiated in NCU over and against UCN.

For CUC,

$$E(t) = \frac{1}{p}t_1 + \frac{p-1}{p}\cdot\frac{q}{Ppq-1}\cdot\sum_{i'=2}^{Ppq} H(q-1;\, Ppq-2,\, Ppq-i',\, q-1)t_{i'}, \qquad (2)$$

and for CNC,

$$E(t) = \frac{1}{p}t_1 + \frac{p-1}{p}\cdot\frac{Pq}{Ppq-1}\cdot\sum_{i'=2}^{Ppq} H(Pq-1;\, Ppq-2,\, Ppq-i',\, Pq-1)t_{i'}. \qquad (3)$$

The value of Pr(t_1) is $1/p$ in each case. As sets of t_1 and Δt_i also are shared across the structures, differences in E(t) necessarily emanate from unequal values of Pr($t_{i'}$). Values of Pr($t_{i'}$) are plotted against i' in Figure 1, for a representative set of parameter values, $p = 2$, and $q = 4$, with P varying from 2 to 5. Benefits of pre-disclosure of bin

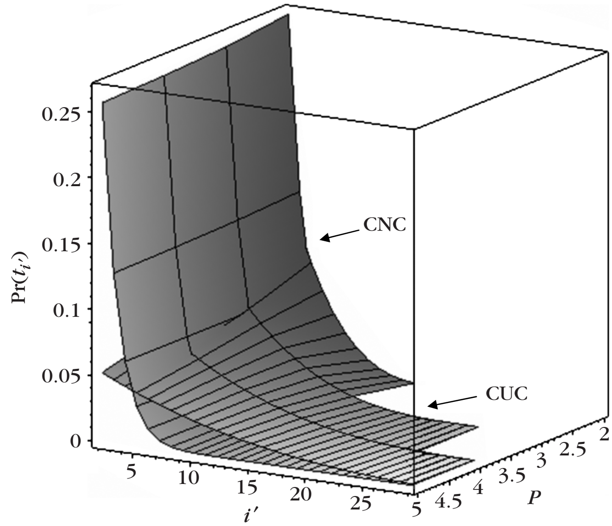

Figure 1. Differences in probabilities of occurrence of $t_{i'}$, as plotted against i', for scenario structures illustrating adverse effects of uncertainty U on threat reduction: CUC, denoting free choice over bin sets C, uncertainty of to-be-assigned bin U, and free choice over bin elements C within the assigned bin versus CNC, whose only difference from CUC is no uncertainty of bin assignment N.

assignment attending CNC, relative to its absence in CUC, are channelled specifically through a comparatively higher Pr($t_{i'}$) for lower $t_{i'}$ values, and vice versa.

The following scenario-structure mechanism is responsible for this effect. In the case of CUC, relatively lower values of $t_{i'}$ are viable if and only if embedded, bin-set wise, with t_1 (and not in t_1's bin itself); this restriction does not apply in the case of CNC. Here, release of that contingency, effected through foreknowledge of a non-assignment of t_1's bin, frees up choice of $t_{i'}$, given its own bin assignment, and competitively low value.

Turning to the comparison of scenario structures NCU and UCN, E(t) for NCU is

$$\left[\frac{1}{P}\left(\frac{1}{q}t_1+\frac{q-1}{q}\bar{t}_{i'}\right)\right]+\frac{P-1}{P}\cdot\frac{pq}{Ppq-1}\cdot\sum_{i'=2}^{Ppq}\left[H(pq-1;Ppq-2,Ppq-i',pq-1)\frac{1}{q}\right.$$
$$\left.+\left\{(1-H(pq-1;Ppq-2,Ppq-i',pq-1))\cdot\frac{q-1}{pq-1}\cdot\frac{1}{q}\right\}\right]t_{i'},\tag{4}$$

where $\bar{t}_{i'}$ is the mean of $Ppq-1$ values of $t_{i'}$. Note that the expression in braces conveys potential encounter of $t_{i'}$, even if it is not the lowest among the pq values of $t_{i'}$ in an assigned bin set containing itself, but not t_1. The probability of engaging such a $t_{i'}$ in this case is the complement of the probability of its instead being the lowest, multiplied by the probability of being bin-wise embedded with what actually is the bin set's lowest $t_{i'}$, $(q-1)/(pq-1)$, in turn along with the probability of its being the assigned element from this element-nesting bin $1/q$. All told, by equation (4), $t_{i'}$ potentially is engaged, (a) if it is bin-wise embedded with t_1, which is implemented in first square-bracketed expression in (4); (b) if it is the lowest among the pq values of $t_{i'}$ in a bin set not containing t_1, which is implemented in the expression outside braces in the larger square-bracketed expression; and (c) if it is not the lowest of the above pq values of $t_{i'}$, but nevertheless is bin-wise embedded with the lowest of these values, which is implemented in the expression in braces. These mutually exclusive conditions together imply that each $t_{i'}$ stands to be engaged, regardless of i'. In contrast, in the case of UCN (for which E(t) is given in Table 2), by prevailing assumptions, the $p - 1$ highest of the Ppq elements t_i have no chance of engagement.

A representative plot of Pr($t_{i'}$) against i' is presented in Figure 2 ($P = p = q = 4$). As with CUC, as opposed to CNC, the higher E(t) associated with NCU, as opposed to UCN, results from lower Pr($t_{i'}$) attending lower $t_{i'}$ values, and vice versa. Moreover, the above asymptote of NCU expresses the inclusive eligibility of $t_{i'}$. Roughly, otherwise non-qualifying $t_{i'}$ values are endowed with a stochastic 'faint threat', whereby no matter the level of threat, by the current contingencies, an element encounter can nevertheless be released upon the 'stress negotiator'. There continues to be, in other words, a 'lingering uncertainty' imposed by NCU, such that high-threat elements remain (probabilistically) lurking, by sheer dent of embedding among more benign elements (i.e., conditions (a) and (c), above).

5. Discussion

5.1. Inter-attribute associations

Associations between indices of two mathematically and empirically separable stressors (Kukde & Neufeld, 1994; Morrison *et al.*, 1988) were shown by the simulation results to be ubiquitous over decisional-control parameters and structure complexity.

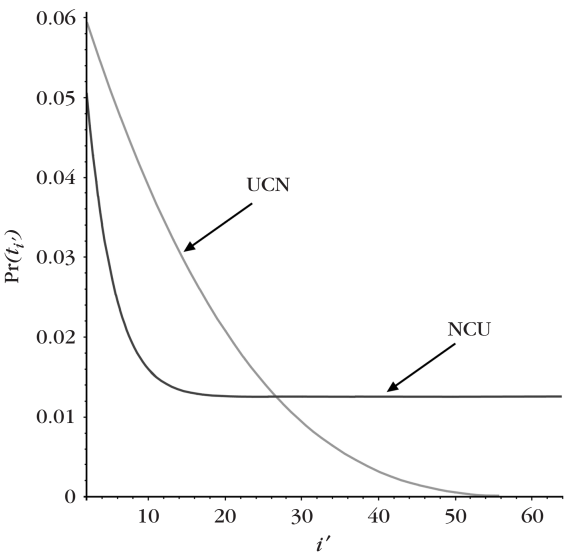

Figure 2. Differences in probabilities of occurrence of $t_{i'}$, as plotted against i', for scenario structures illustrating adverse effects of positioning of uncertainty U in relation to free choice C on threat reduction: NCU, denoting no uncertainty of to-be-assigned bin set N, free choice over bins C within the assigned bin set, and uncertainty of assigned elements U within each bin; versus UCN, whose only difference from NCU is the interchange of (un)certainty between bin-set and bin-element assignment.

Prevailing threat as a statistic of the environment (cf. Movellan & McClelland, 2001) after maximizing decisional control is 'exogenous to the stress negotiator'; *cognitive work* is harnessed to expected processing demands endogenous to the DM. Subjective trade-off preferences between these stressors are a candidate source of individual differences in propensity towards decisional control (Morrison *et al.*, 1988; Neufeld, 1999c; Shanahan & Neufeld, 2008).

Formal expressions of cognitive work (Townsend & Ashby, 1978, 1983; Townsend & Wenger, 2004a, 2004b; Wenger & Townsend, 2000; see also clinical implementation of Neufeld, Townsend, & Jetté, 2007; and stress significance, Neufeld, 1990) present themselves as a cognitive-load translation of the defined scenario structure's information-processing demands OSS. Scanning for a scenario structure's potential encounter of least threat tenably is represented as a dynamical process whose completion is stochastically distributed over time (for mathematical underpinnings, see Townsend & Nozawa, 1995). Such a process, in turn, can be conceived as comprising constituent operations, or subprocesses, OSS in number. Assuming process latency T is continuously distributed, the amount of cognitive work accomplished by time T is estimable as $-\ln(1 - F(T))$, where $F(T)$ is the distribution's cumulative distribution function, or probability of process completion at or before T (Townsend & Ashby, 1983).

The underlying architecture of the subprocess-dispatching system – notably its serial, parallel (including variants thereof), or hybrid organization of subprocess completion (Townsend & Ashby, 1983) – in principle can be identified through contemporary mathematical diagnostics of mental architecture, as applied to process-latency distributions (indeed, as can the exhaustive search implied by a maximizing choice strategy; Townsend & Wenger, 2004b). Existing empirical latency data bearing on process magnitude is not out of keeping with the current definition of decisional-control processing demands as OSS (Kukde & Neufeld, 1994; Morrison *et al.*, 1988; see also below). More fine-grained analysis and associated quantitative signatures of processing architecture, and collateral properties, nevertheless are tenable for characterization of the cognitive load imposed by the present cognition-intensive avenue of coping. In this way, the theoretical prototyping of decisional control potentially dovetails with selected measurement technology of contemporary cognitive science.

The reciprocal relation between the separable stressors varying with decisional control was present for all simulation inputs. The model thus withstood this version of 'model sensitivity analysis', its predicted qualitative patterns of attribute (co)variation being robust across variation in structure-parameter values. It also withstood 'model generalization analysis', in that the parameter landscape spanned by the predicted patterns was extensive, and they withstood the change in hierarchical structure complexity.[2]

We now turn our attention to the association between $E(t)$ and its tendered surrogate $Pr(t_1)$. Note first that $Pr(t_1)$ is immediately tractable, being conceptually and computationally straightforward. Moreover, $Pr(t_1)$ is general in that, unlike $E(t)$, it transcends numerical specifics. This index of threat reducibility also is related to the quantitative definition of decisional control by a constant of proportionality, in this way lending construct validity to that definition, and vice versa. Its validity as a gauge of threat reducibility nevertheless also is adjudged by $Pr(t_1)$'s association with $E(t)$ as a normative model formal benchmark. Sanguinity is encouraged by $E(t)$'s scaling, by the complement of $Pr(t_1)$, the mean of the complementing set of t_i values $\{t_2, t_3, \dots, t_{(P)pq}\}$ in a subset of the examined structures (Table 2; Appendix B). Remaining structures, however, would be expected to perturb this otherwise perfect association, and 'informational completeness' of $\Pr(t_1)$. The simulations indicate a 'high moderate' average association between $\Pr(t_1)$ and $E(t)$. However, they also signal that confidence in $\Pr(t_1)$ as a substitute for $E(t)$ is justified in certain parameter regions more than others. Given the scalar impact of threat levels confirmed in the analyses, bivariate correlation values between $\Pr(t_1)$, $E(t)$, and OSS can be determined independent of levels of (equally spaced) element-wise threat. Authoritative tables reporting degree of association between these quantities for given p, q pairs and P, p, q triplets can be consulted for regions of specific interest (available in supporting online materials, http://publish.uwo.ca/~mshanah).

5.2. Candidate sources of individual differences in propensity towards decisional control

The quantified decisional-control structure attributes $\Pr(t_1)$ and OSS, and their documented positive relation across structures, usher in an integration with selected formal models of preference and choice befitting this very structure composition (e.g., Tversky's EBA, 1972a, 1972b; Batsell *et al.*, 2003; see also Busemeyer, Forsyth, & Nozawa, 1988; Saari, 2005).[3] Made available in principle are estimated subjective utilities of reduction in each constituent stressor, and description and prediction of preference behaviours regarding structures in which the stressors are embedded (Morrison *et al.*, 1988). Fostering this integration, developments presented here have ascertained the generality of the above positive relation – also known as 'incompatibility of criteria' (Tversky, 1969; Tversky & Russo, 1969) – across both structure parameters and complexity.

A tenable source of individual differences in subjective trade-off of the separable stressors is differential facility with cognitive functions subserving decisional-control's requisite predictive judgements ('cognitive measurement technology', above). Alongside is differential susceptibility to adverse effects of stress itself, on efficiency of such functions, as they take place in stressing environments (Neufeld, 1994, 1999a; Neufeld & McCarty, 1994; Neufeld *et al.*, 2007; see also Ferguson, 2004; Ferguson & Bibby, 2002). Pursuant to individual-difference extremes, the present developments offer a formal platform for the potential analysis of compromised stress resolution associated with cognitive deficit in clinical disorders, such as schizophrenia (e.g., Neufeld, 2007; Townsend & Neufeld, 2010; cf. Kirschbaum, Pirke, & Hellhammer, 1993).[4]

5.3. Choosing into uncertainty

A marked feature of the across-structure configuration of threat expectation is the undermining of threat attenuation through strategic choice, when condition U is located in subordinate positions of a hierarchy structure. Following a sequence of empirical studies incorporating behavioural and psychophysiological measures of stress activation, Paterson and Neufeld (1995) observed that heightened evidence of stress was identified with conditions of 'blind selection' and increased information-processing

[3] Empirical support for this extension has taken the form of substantial EBA model fitting and testing, with scenario properties $\Pr(t_1)$ and OSS forming the basis of EBA prescribed attributes of the decisional-control scenario preference options (Benn, 2001; Benn & Neufeld, 1996, 2010).

[4] Note that individualized assessment of faculties entering into visual- and memory-search processes instigated by decisional control stands to be aided by contemporary implementation of Bayesian measurement technology (e.g., Neufeld *et al.*, 2010; Rouder, Sun, Speckman, Lu, & Zhou, 2003).

load under time pressure. Tendered here are quantitative mechanisms highly coherent with these observations: under the above circumstances of condition U, lower threat encounters have a demonstrably reduced probability of occurrence, and the opposite for higher threat encounters; moreover, there can remain a residual viability of a scenario structure's elements of highest threat (Figure 2). Such intersection with experimental results illustrates how quantitative normative models of stress and coping may coalesce with empirical findings, in the service of validating the first and explaining the second.

5.4. Relations to allied modelling

The present developments convey prototypes of decisional-control transactions, falling into the category of a quasi-stochastic static model – $\Pr(t_i)$, t_i, and $E(t)$ are probabilistic, but RSS and OSS are deterministic (Busemeyer & Townsend, 1993). Upon encountering natural or laboratory instantiations, individuals are apt to array themselves in terms of differential predilection to decisional-control opportunities, according to facility with requisite cognitive functions and personality traits. Moreover, individual dispositions may realign upon repeated encounters; proficiency in requisite cognitive functions may increase, as may propensity to engage in available decisional control with cumulative experiences of its threat-reducing pay-off.

Modelling addressed to other game-related domains entailing risky, uncertain choice, notably that of altruism and public goods provision, on the other hand has addressed dynamical changes and individual differences (e.g., Anderson, Goeree, & Holt, 1998; Itaya & Shimomura, 2001). Models of this nature may productively inform future developments of the form of modelling presented in this paper.

Certain nonlinear dynamical systems ('chaos-theoretic') modelling meanwhile has expressly invoked, as an 'order dimension', decisional-control coping, whose continuous dynamics, along with those of collateral variables (e.g., cognitive proficiency and engagement propensity) are expressed in terms of coupled differential equations (Neufeld, 1999a). Moreover, emergent discontinuous dynamics comprise system transmogrification, occurring to continuous system-parameter changes (Levy *et al.*, 2010; Levy, Yao, Vollick, McGuire, & Neufeld, 2006; Yao *et al.*, 2006). Unlike the case with dynamic linear public goods games, however, although part and parcel of a quantitative dynamical system, decisional control here does not have formal dynamics interwoven into its game-like structure itself.

The present developments also depart from other related formulations, in compliance with certain sui generis stress-and-coping exigencies. Decision field theory (Busemeyer & Townsend, 1993), for example, addresses choice alternatives composed of probabilities of varying financial pay-offs, and is fully stochastic (see also Birnbaum, 2008; Johnson & Busemeyer, 2005). Also, contra these formulations, the present developments stress the intrinsically reciprocal relations between posited 'commodities', threat reduction, and reduction in cognitive load.

The formal systematization of the relations between threat reduction and cognitive load allows for a new opportunity to instantiate a transactional approach to stress and coping (Lazarus & Folkman, 1984). A transactional approach highlights the mutual influence of DM and environment, in an iterative format, wherein the exercise of decisional control at time 1 alters the threat statistics of the environment for the exercise of decisional control at time 2. This Markov chain-type concatenation of decisional scenarios would incorporate the intransigent formal relations of the relevant decisional-control situational attributes, while allowing for the specific values to be dynamically influenced by previous choices. If a specific progression of numerous threat-reducing decisions were considered a dynamic process (as per Lazarus & Folkman, 1984), the present model allows for the attribute values generated for each self-contained 'decision cell' to have a high degree of formal integrity, strengthening the validity of simulations of transactional stress processes.

Whereas Lazarus and Folkman's (1984) theory of cognitive appraisal includes an emotional component in the perception of stress, they refer to Janis and Mann (1977) as developing a more purely cognitive approach to stress resolution through evaluation of courses of action. Our model is in this vein, and provides a normative basis from which to assess any particular DM's efficacy in enacting available decisional control for stress reduction. For example, suppose a dedicated shift manager is promoted to a new level of responsibility within a factory, and is now responsible for safety in the entire plant. The decision-making efficiency of this individual is now more critical. The set of heuristics that are suitable for a shift manager may not be optimal for a director of plant safety. One avenue indicated by our model might be to assess whether the DM is aware of the general efficiency of the heuristic of probability of access to the least threatening option as an index of threat reducibility. The DM could be trained to spot this configuration in various scenarios the employees face in the plant, and increase this value by decreasing the set size at nodes with constraints on choice. The new safety director could be assessed for the degree to which he or she can evaluate the optimal stress-reduction structure to give employees and assessed for the concomitant skill of normative threat negotiation in a given scenario. A particular application of this training could be that if workers are to be given free choice of shift but externally assigned what product to make on a given shift (scenario structure CN), minimizing the number of possible product assignments (min q) will most likely maximize the worker's ability to negotiate their risk of injury, job stress level, or general aversiveness of their work ($\mathrm{E}(t) \propto \Pr(t_1)$; $\Pr(t_1) = 1/q$). A further benefit would be that individual workers, in exercising decisional control along individual differences patterns, may have differing evaluations of the aversiveness or very general 'threat level' of a given assignment, thus ensuring some distribution of labour across a variety of tasks by allowing individual predilection some scope of action in decision making.

The job demand-control model (Karasek, 1979) uses a stress and coping approach in the workplace and introduced the notion of job strain as the combination of high job demands and low job decision latitude. There is an interesting relation between the positional effect of uncertainty within a decision hierarchy in our model and the approach to assessing job decision latitude as defined by Karasek. Consonant with Karasek, our model supports the idea that low decision latitude will hamper threat reduction by limiting available decisional control. Further to this observation, however, when mixed decisional structures (some choice, some external assignment) within an organization are set up in a framework such as UC, for example, considerable advantage may accrue to individual employees when it comes to objective threat-reduction capabilities, over a CU arrangement. Conversely, some job decision latitude, although appearing to be devolving stress negotiation to a local DM, may be largely illusory in the threat-reduction opportunity it confers. Overall, however, any degree of choice is superior to pure external assignment (N or U conditions only).

A final comment on this application of the model deals with underlying assumptions. While the U and N assignments are assumed to be random, there is a way in which external assignment in practice would imitate this, and a way in which it would not. Random assignment is a valid assumption to the extent that employees are treated like a capital resource, the assignment of whom (of employee to task, nested in task location, for example), is independent of the employee's own considerations. Thus the external assignment is 'blind' to the employee's threat-reduction aims. On the other hand, the random assignment assumption becomes weaker to the extent that management of morale, employee preference, and individual ability profile is incorporated in decision making. Nonetheless, assuming that the DM (employee or other self-motivated agent) is ensuring their own welfare, while the external assignment is done for considerations largely apart from the DM's threat reduction, the assumption of equal probability of assignment within N or U conditions is tenable.

5.5. Empirical touchstones and further research

Formal models of stress and coping may be used to test theory according to their analytic derivations and collateral computer computations. As indicated throughout, they also prescribe empirical observations. Below, we highlight the explanatory power of the model in predicting empirical response patterns in data collected with psychophysiological and behavioural measures. We go on to suggest a broader scope for further principled investigation in decision-making science, specifically within the learned helplessness paradigm (Seligman, 1975) and with respect to the quantitative assessment of individual differences in disposition to implement decisional control. Most broadly, the decision-making paradigms of *maximizing* (Janis & Mann, 1977) and *satisficing* (Simon, 1955) may be profitably rounded out to include 'simplifying' (cf. Paquette & Kida, 1988; Wright, 1975), wherein DM-driven artificial relinquishment of information-processing and decision-making demands is prized most highly, at the cost of accepting an event-occurrence threat level that is fully externally driven.

The formally educed structure of decisional control stipulates that designated measures can selectively track its quantified properties, of adverse-event threat and information-processing demands. Such measures include, for example, facial electromyography (EMG). Lip and chin (*orbicularis oris* and *digastricus*) myoactivity are deemed as sensitive and specific to covert information processing, while forehead (*frontalis*) EMG is held to selectively indicate subjective threat of adverse-event occurrence (reviewed in Kukde & Neufeld, 1994). These measures therefore should vary in opposite directions across conditions of experimental stress varying in availability of decisional control.

Such fractionation is presented in Figure 3, for a subset of first-order pairs of choice conditions. Participants were presented with two sets of alphabetic letters, under choice conditions C or N governing selection possibilities of letter sets, combined with conditions C, U, or N governing selection possibilities of letters nested within sets. Facial EMG, displayed in the figure, was obtained while participants selected an available letter, which according to preliminary rounds of probability learning (see segment on unpredictability in the supplementary online document), was associated with minimal likelihood of an adverse event (a burst of safe but aversive white noise; full details are available in Kukde & Neufeld, 1994).

The contour of response latencies, presented in Figure 4, also essentially conforms to model expectations across the combinations of choice conditions. As well, the direction of intra-trial subjectively reported stress (accompanied by skin-conductance and cardiac measures) paralleled that of lip and chin EMG, while that of adverse-event probability, previously paired with the selected letter, resembled the path of forehead EMG (all generally replicating findings previously reported in Morrison *et al.*, 1988).

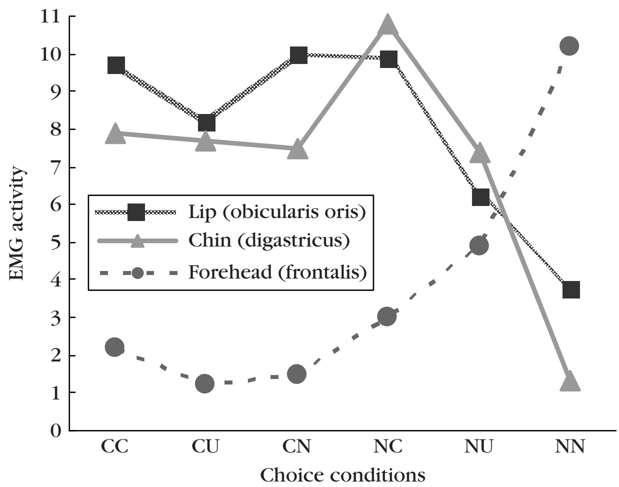

Figure 3. Facial EMG activity across a subset of first-order combinations of decisional-control choice conditions. Letter pairs on the abscissa indicate choice condition over letter sets, choice condition over letters nested within sets; C, free choice over constituents; N, no uncertainty as to constituent assignment; U, uncertainty of to-be-assigned constituent. Values for lip and forehead EMG are collapsed over two versus four letters per each of two-letter sets, for each pair of choice conditions, and across the first versus second (randomly interspersed) trial of each of the 12 (6×2) combinations of choice conditions and letter-set size; those for chin EMG are for the first trial of the two-letter set size presentations (adapted from Kukde & Neufeld, 1994).

These results illustrate how formal developments can give rise to experimental paradigms and empirical predictions. The subset of choice conditions in the present example was limited according to laboratory constraints. However, other feasible subsets from the current set of 9 first-order or 27 second-order combinations of choice conditions, along with any number of values for P, p, and q, are available for similar experimental implementation.

Predictions concerning other measurement fractionations also are available. Signatures of experienced threat versus challenge have been derived from cardiac and vascular response patterns (Blascovich, 2008). Placing a premium on threat reduction, over and against cognitive-load reduction, based on individualized parameters of formal preference-choice models (Sections 1 and 5.2) is predicted to be positively associated with evidence of challenge versus threat (Blascovich, Seery, Mugridge, Norris, & Weisbuch, 2004) – specifically where choice conditions afford increased decisional control (currently under study in our laboratory).

Developments presented here provide substantial numerical-point values from an expansive charting of scenario parameters, and decisional-control property inter-relations, affording considerable fodder for empirical predictions. They also throw light on existing findings (cf. Haig, 2008) from laboratory (e.g., Paterson & Neufeld, 1995) and field settings (Wells & Matthews, 1994).

Additional empirical predictions bear on barriers to threat reducibility stemming from the introduction and positioning of uncertainty U. Choice condition combinations predicted to generate increased evidence of adverse-event threat, as compared to more uncertainty-benign conditions, are specified in Sections 4.3.1 and 4.3.2.

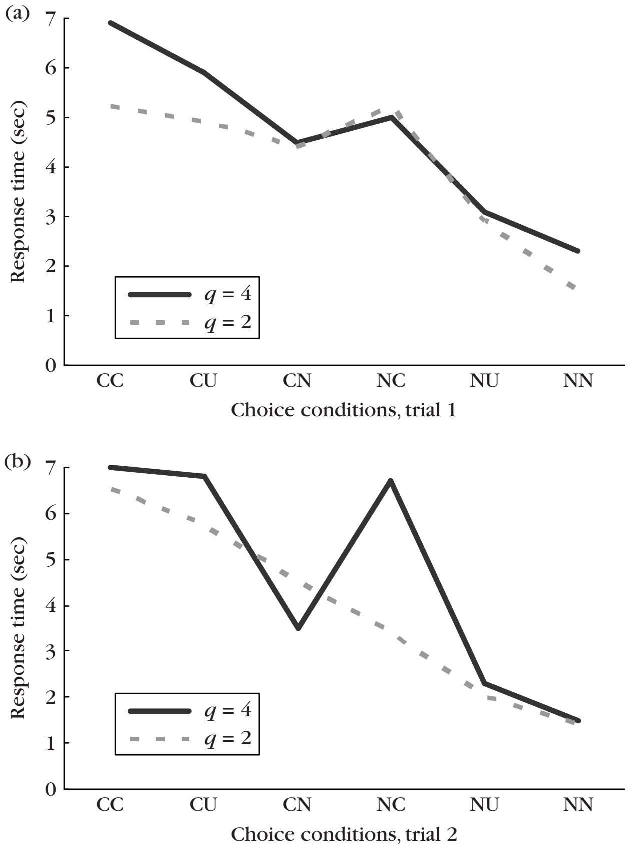

Figure 4. Response latencies to letter selection across a subset of first-order combinations of decisional-control choice conditions, for each combination of choice conditions and element-set size q, separately for the first and second (randomly-interspersed) trial of the 12 (6×2) combinations of choice conditions and set size. Upper-case letter pairs on the abscissa correspond to those described in Figure 3 (adapted from Kukde & Neufeld, 1994).

A novel prediction on a broader scale can include the application of the positioning effect of uncertainty U to the learned helplessness paradigm (Seligman, 1975). The learned helplessness paradigm describes how an organism can learn that there are no actions that will reduce the threat statistics of their environment. However, this learning becomes overgeneralized, and is incorrectly applied to negate the potential of action in one's circumstances to improve one's lot in terms of ambient threat. One of the 'actions' which may unfortunately be curtailed is the cognitive exercise of decisional control. Thus, individuals may generalize from previous experience that most situations are, in the terms of this model, NN or UU – fully externally determined, when they may not be. In a more subtle way, there may even be some form of partial helplessness discouragement if an individual is in a C-with-subordinate-U situation, wherein apparent advantage due to C is in fact somewhat or even mostly illusory, due to a subordinate U. Individuals with depression are among Seligman's principal groups of interest for application of learned helplessness. Such a person may generate considerable self-reproach, or experience reproach from others, for not improving his or her situation through decision and action. However, if the decisional and informational structure of the situation is such that previous attempts at coping have been made within decisional scenarios with little or no objective opportunity for threat reduction, the 'learned helplessness' of such an individual is quite legitimate. Further and more comprehensive mapping of how to structure decisional scenarios within individual lives to foster the opposite of learned helplessness (e.g., 'optimism' or 'resilience' in Reivich, Gillham, Chaplin, & Seligman, 2005; 'personal agency' in Bandura, 2006) stand to be informed by this model's quantification of decisional-control's threat-reducing efficacy.

Following the reasoning of the application of decisional control to a learned helplessness paradigm, the individual differences in predilection to exercise decisional control are quite relevant to explore with a view to adding an instrument or two to the toolbox of personality measurement in psychology. Specifically, the potential exists to artificially move from a scenario with free choice C, for example CN, to an NN scenario by simply making a random selection where free choice is available. The pay-off is reduced information-processing demand. For individuals with emotional distress and/or cognitive impairment, this may become the strategy of choice. This type of decision-making strategy, placed on a spectrum from *maximizing* (exhaustive search for maximum benefit), through *satisficing* (search sufficient for tangible advantage), may be termed *simplifying* (no search and acceptance of ambient environmental threat statistics, in return for escaping the cognitive demands of negotiating threat reduction).

5.6. Concluding comments

The current game-like formulation of decisional control comprises a normative theoretical model of environmental exigencies underlying reduction in threat of adverse events. As with normative models generally, it affords a template of statistically optimal behaviours ('maximizing' decision and choice strategies) against which descriptive models (Edwards, 1988; Over, 2004) in principle can be fashioned and/or selected, and from which the nature and degree of descriptive-model departure can be ascertained (e.g., Maddox & Filoteo, 2007; Neufeld *et al.*, 2007).

Heuristic assets include predictions of responses tracking quantitatively dissociable sources of stress activation. They also include: delineation of psychometrically measurable individual differences in response patterns, and sources of such differences associated with decisional control implementing cognitive faculties, and with susceptibility to stress effects on cognitive efficiency; connections to formal models of decision and choice dovetailing with the structure of decisional-control attributes, and predicting differential preference for decisional-control coping, and realization of its advantages; and connections to complementing forms of formal modelling of stress- and decisional-control-based coping, notably those of nonlinear dynamical systems theory. Delineated are exacerbating effects, on compromised threat reducibility, of subordinate locations of outcome uncertainty within systematically constructed hierarchical scenario structures. Also brought into play are situation properties potentially significant for psychological stress that bear on adverse-event predictability (event-variance considerations).

The amalgam of analytic and simulation developments may serve as a prototype for extensions transcending the present assumptive framework. Such extensions invite similar quantitative conversions of verbal axioms for discerning networks of stress-coping attributes.

Acknowledgements

Simulations reported in this paper formed part of the first author's master's thesis in clinical psychology (Shanahan, 2007), supervised by the second author. We thank Lorrie Lefebvre for her patient and diligent work in helping prepare this manuscript.
This research was supported by the Social Science and Humanities Research Council of Canada, in the form of a Canada Graduate Scholarship to the first author, and an operating grant to the second author.

References

Anderson, S. P., Goeree, J. K., & Holt, C. (1998). A theoretical analysis of altruism and decision error in public goods games. *Journal of Public Economics*, *70*, 297–323.

Averill, J. R. (1973). Personal control over aversive stimuli and its relationship to stress. *Psychological Bulletin*, *80*, 286–303.

Averill, J. R. (1979). Anger. In H. E. Howe, Jr., & R. A. Dienstbier (Eds.), *Nebraska Symposium on Motivation* (Vol. 26, pp. 1–80). Lincoln: University of Nebraska Press.

Bandura, A. (2006). Toward a psychology of human agency. *Perspectives on Psychological Science*, *1*(2), 164–180.

Batsell, R. R., Polking, J. C., Cramer, R. D., & Miller, C. M. (2003). Useful mathematical relationships embedded in Tversky's elimination by aspects model. *Journal of Mathematical Psychology*, *47*, 538–544.

Benn, K., & Neufeld, R. W. J. (2010). *Formal modelling of differential preference for decisional control over stress*. Manuscript in preparation.

Benn, K. D. (2001). *Formal modelling of decisional-control preferences and mechanisms underlying stress negotiation*. London: Faculty of Graduate Studies, University of Western Ontario.

Benn, K. D., & Neufeld, R. W. J. (1996, August). *Decisional control under stress*. Poster Session presented at the 104th Annual Convention of the American Psychological Association, Toronto, Canada.

Birnbaum, M. H. (2008). New paradoxes of risky decision making. *Psychological Review*, *115*, 463–501.

Blascovich, J. (2008). Challenge and threat. In A. J. Elliott (Ed.), *Handbook of approach and avoidance motivation* (pp. 431–445). New York: Psychology Press.

Blascovich, J., Seery, M. D., Mugridge, C. A., Norris, R. K., & Weisbuch, M. (2004). Predicting athletic performance from cardiovascular indexes of challenge and threat. *Journal of Experimental Social Psychology*, *40*, 683–688.

Booth, P. I. (1985). A geometric interpretation of the effects of anxiety on test performance. *Mathematical Intelligencer*, *7*, 56–63.

Burger, J. M., & Cooper, H. M. (1979). The desirability of control. *Motivation and Emotion*, *3*, 381–393.

Busemeyer, J. R., Forsyth, B., & Nozawa, G. (1988). Comparisons of elimination by aspects and suppression of aspects: Choice models based on choice reaction time. *Journal of Mathematical Psychology*, *32*, 341–349.

Busemeyer, J. R., & Townsend, J. T. (1993). Decision field theory: A dynamic-cognitive approach to decision making in an uncertain environment. *Psychological Review*, *100*, 432–459.

Cacioppo, J. T., Petty, R. E., & Kao, C. F. (1984). The efficient assessment of need for cognition. *Journal of Personality Assessment*, *48*, 306–307.

Connor-Smith, J. K., & Flachsbart, C. (2007). Relations between personality and coping: A meta-analysis. *Journal of Personality and Social Psychology*, *6*, 1080–1107.

Edwards, W. (1998). Tools for and experiences with Bayesian normative modeling. *American Psychologist*, *53*, 416–428.

Ferguson, E. (2004). Conscientiousness, emotional stability, perceived control and the frequency, recency, rate and years of blood donor behaviour. *British Journal of Health Psychology*, *9*, 293–314. doi:10.1348/1359107041557011

Ferguson, E., & Bibby, P. A. (2002). Predicting future blood donor returns: Past behavior, intentions, and observer effects. *Health Psychology*, *21*, 513–518.

Greco, V., & Roger, D. (2001). Coping with uncertainty: The construction and validation of a new measure. *Personality and Individual Differences*, *31*, 519–534.

Guastello, S. J. (1992). Accidents and stress-related health disorders: Forecasting with catastrophe theory. In J. C. Quick, J. J. Hurrell, & L. M. Murphy (Eds.), *Work and well-being: Assessment and interventions for occupational mental health* (pp. 252–269). Washington, DC: American Psychological Association.

Haig, B. D. (Ed.) (2008). Scientific method, abduction, and clinical reasoning. *Journal of Clinical Psychology*, *64*, 1013–1127.

Holahan, C. J., Holahan, C. K., Moos, R. H., Brennan, P. L., & Schutte, K. K. (2005). Stress generation, avoidance coping, and depressive symptoms: A 10-year model. *Journal of Consulting and Clinical Psychology*, *73*, 658–666.

Itaya, J., & Shimomura, K. (2001). A dynamic conjectural variations model in the private provision of public goods: A differential game approach. *Journal of Public Economics*, *81*, 153–172.

Janis, I. L., & Mann, L. (1977). *Decision making: A psychological analysis of conflict, choice, and commitment*. New York: Free Press.

Johnson, J. G., & Busemeyer, J. R. (2005). A dynamic, stochastic, computational model of preference reversal phenomena. *Psychological Review*, *112*, 841–861.

Karasek, R. A., Jr. (1979). Job demands, job decision latitude, and mental strain: Implications for job redesign. *Administrative Science Quarterly*, *24*(2), 285–308.

Kirschbaum, C., Pirke, K., & Hellhammer, D. H. (1993). The 'Trier Social Stress Test': A tool for investigating psychobiological stress responses in a laboratory setting. *Neuropsychobiology*, *28*, 76–81.

Kukde, M. P., & Neufeld, R. W. J. (1994). Facial electromyographic measures distinguish covert coping from stress response to stimulus threat. *Personality and Individual Differences*, *16*, 211–228.

Lazarus, R. S., & Folkman, S. (1984). *Stress, appraisal, and coping*. New York: Springer.

Lees, M. C., & Neufeld, R. W. J. (1999). Decision-theoretic aspects of stress arousal and coping propensity. *Journal of Personality and Social Psychology*, *77*, 185–208.

Levy, L., Yao, W., Vollick, D., McGuire, C. G., Hay, J., & Neufeld, R. W. J. (2010). *A substantively spawned six-dimensional nonlinear dynamical systems model of stress and coping produces substantively significant bifurcations, including the Feigenbaum progression to chaos*. Manuscript in preparation.

Levy, L. R., Yao, W., Vollick, D., McGuire, C. G., & Neufeld, R. W. J. (2006, July). *Nonlinear dynamics of stress, coping and related variables*. Poster presented to the Society for Mathematical Psychology Annual Meeting, Vancouver, Canada.

Maddox, W. T., & Filoteo, J. V. (2007). Modeling visual attention and category learning in amnesiacs, striatal-damaged patients, and normal aging. In R. W. J. Neufeld (Ed.), *Advances in clinical-cognitive science: Formal modeling and assessment of processes and symptoms* (pp. 113–146). Washington, DC: American Psychological Association.

Monroe, S. M. (2008). Modern approaches to conceptualizing and measuring human life stress. *Annual Review of Clinical Psychology*, *4*, 33–52.

Morrison, M. S., Neufeld, R. W. J., & Lefebvre, L. A. (1988). The economy of probabilistic stress: Interplay of controlling activity and threat reduction. *British Journal of Mathematical and Statistical Psychology*, *41*, 155–177.

Movellan, J., & McClelland, J. L. (2001). The Morton–Massaro law of information integration: Implications for models of perception. *Psychological Review*, *108*, 113–148.

Neufeld, R. W. J. (1982). On decisional processes instigated by threat: Some possible implications for stress-related deviance. In R. W. J. Neufeld (Ed.), *Psychological stress and psychopathology*. New York: McGraw-Hill.

Neufeld, R. W. J. (1990). Coping with stress, coping without stress, and stress with coping: On inter-construct redundancies. *Stress Medicine*, *6*, 117–125.

Neufeld, R. W. J. (1994). *Theoretical stress and stress-proneness effects on information processing in light of mathematical models of stochastic processes*. Department of Psychology Research Bulletin No. 720. London: University of Western Ontario.

Neufeld, R. W. J. (1999a). Dynamic differentials of stress and coping. *Psychological Review, 106*, 385–397.

Neufeld, R. W. J. (1999b). Formally-inferred properties of decisional stress-control. *Research Bulletin Number 756*. Department of Psychology, University of Western Ontario.

Neufeld, R. W. J. (1999c). *Quantitative schemata of decisional stress-control*. Paper presented at the 32nd Annual Meeting of the Society for Mathematical Psychology, Santa Cruz, CA, June.

Neufeld, R. W. J. (2007). On the centrality and significance of encoding deficit in schizophrenia. *Schizophrenia Bulletin*, *33*, 982–993.

Neufeld, R. W. J., Boksman, K., Vollick, D., George, L., & Carter, J. R. (2010). Stochastic dynamics of stimulus encoding in schizophrenia: Theory, testing and application. *Journal of Mathematical Psychology*, *54*(1), 90–108. doi:10.1016/j.jmp.2009.04.002

Neufeld, R. W. J., & McCarty, T. (1994). A formal analysis of stressor and stress-proneness effects on basic information processing. *British Journal of Mathematical and Statistical Psychology*, *47*, 193–226.

Neufeld, R. W. J., & Paterson, R. J. (1989). Issues concerning control and its implementation. In R. W. J. Neufeld (Ed.), *Advances in the investigation of psychological stress* (pp. 43–67). New York: Wiley.

Neufeld, R. W. J., Townsend, J. T., & Jetté, J. (2007). Quantitative response time technology for measuring cognitive-processing capacity in clinical studies. In R. W. J. Neufeld (Ed.), *Advances in clinical cognitive science: Formal modeling and assessment of processes and symptoms* (pp. 207–238). Washington, DC: American Psychological Association.

Over, D. (2004). Rationality and the normative/descriptive distinction. In D. J. Koehler & N. Harvey (Eds.), *Blackwell handbook of judgment and decision making* (pp. 3–18). Malden, MA: Blackwell.

Paquette, L., & Kida, T. (1988). The effect of decision strategy and task complexity on decision performance. *Organizational Behavior and Human Decision Processes*, *41*(1), 128–142.

Paterson, R. J., & Neufeld, R. W. J. (1995). What are my options: Influences of choice availability on stress and the perception of control. *Journal of Research in Personality*, *429*, 145–167.

Patil, G. G., & Joshi, S. W. (1968). *A dictionary and bibliography of discrete distributions*. New York: Hafner.

Rappaport, A. (1983). *Mathematical models in the social and behavioral sciences*. New York: Wiley.

Reivich, K., Gillham, J. E., Chaplin, T. M., & Seligman, M. E. P. (2005). From helplessness to optimism: The role of resilience in treating and preventing depression in youth. In S. Goldstein & R. B. Brooks (Eds.), *Handbook of resilience in children* (pp. 223–237). New York: Kluwer.

Rouder, J. N., Sun, D., Speckman, P. L., Lu, J., & Zhou, D. (2003). A hierarchical Bayesian statistical framework for response-time distributions. *Psychometrika*, *68*, 589–606.

Saari, D. G. (2005). The profile structure for Luce's choice axiom. *Journal of Mathematical Psychology*, *49*, 226–253.

Seligman, M. E. P. (1975). *Helplessness: On depression, development, and death*. San Francisco: W. H. Freeman.

Shanahan, M. J. (2007). Quantitative properties of the environment and psychological implications of a normative model of decisional control. Master of science thesis. Department of Psychology, University of Western Ontario.

Shanahan, M. J., & Neufeld, R. W. J. (2008). *Modeling cognitive load and threat reduction: Quantitative characteristics of hierarchically-nested decisional-control scenarios*. Paper presented at the 41st Annual Conference of the Society for Mathematical Psychology, Washington, DC, July.

Simon, H. A. (1955). A behavioral model of rational choice. *Quarterly Journal of Economics*, *69*, 99–118.

Solomon, S., Holmes, D. S., & McCaul, K. D. (1980). Behavioral control over aversive events: Does control that requires effort reduce anxiety and physiological arousal? *Journal of Personality and Social Psychology*, *39*, 729–736.

Thompson, S. C. (1981). A complex answer to a simple question: Will it hurt less if I can control it? *Psychological Bulletin*, *90*, 89–101.

Tomaka, J., Blascovich, J., Kibler, J., & Ernst, J. M. (1997). Cognitive and physiological antecedents of threat and challenge appraisal. *Journal of Personality and Social Psychology*, *73*, 63–72.

Townsend, J. T., & Ashby, F. G. (1978). Methods of modeling capacity in simple processing systems. In J. Castellan & F. Restle (Eds.), *Cognitive theory* (Vol. III, pp. 199–239). Hillsdale, NJ: Erlbaum.

Townsend, J. T., & Ashby, F. G. (1983). *Stochastic modelling of elementary psychological processes*. Cambridge: Cambridge University Press.

Townsend, J. T., & Neufeld, R. W. J. (Eds.), (2010). Contributions of mathematical psychology to clinical science and assessment [Special Issue]. *Journal of Mathematical Psychology*, *54*(1).

Townsend, J. T., & Nozawa, G. (1995). Spatio-temporal properties of elementary perception: An investigation of parallel, serial, and coactive theories. *Journal of Mathematical Psychology*, *39*, 321–359.

Townsend, J. T., & Wenger, M. J. (2004a). A theory of interactive parallel processing: New capacity measures and predictions for a response time inequality series. *Psychological Review*, *111*, 1003–1035.

Townsend, J. T., & Wenger, M. J. (2004b). The serial–parallel dilemma: A case study in a linkage of theory and method. *Psychonomic Bulletin and Review*, *11*, 391–418.

Tversky, A. (1969). Intransitivity of preferences. *Psychological Review*, *76*, 31–48.

Tversky, A. (1972a). Choice by elimination. *Journal of Mathematical Psychology*, *9*, 341–367.

Tversky, A. (1972b). Elimination by aspects: A theory of choice. *Psychological Review*, *79*, 281–299.

Tversky, A., & Russo, J. E. (1969). Substitutability and similarity in binary choices. *Journal of Mathematical Psychology*, *6*, 1–12.

Wenger, M. J., & Townsend, J. T. (2000). Basic tools for attention and general processing capacity in perception and cognition. *Journal of General Psychology: Visual Attention*, *127*, 67–99.

Wells, A., & Matthews, G. (1994). Self-consciousness and cognitive failures as predictors of coping in stressful episodes. *Cognition and Emotion*, *8*, 279–295.

Wilson, E. J., MacLeod, C., Mathews, A., & Rutherford, E. M. (2006). The causal role of interpretive bias in anxiety reactivity. *Journal of Abnormal Psychology*, *115*, 103–111.

Wright, P. (1975). Consumer choice strategies: Simplifying vs. optimizing. *Journal of Marketing Research*, *12*(1), 60–67.

Wright, R. A. (1984). Motivation, anxiety, and the difficulty of avoidant control. *Journal of Personality and Social Psychology*, *46*, 214–221.

Yao, W., Yu, P., Essex, C., & Davison, M. (2006). Competitive modes and applications. *International Journal of Bifurcation and Chaos*, *16*(3), 497–522.

Received 3 February 2009; revised version received 22 October 2009

Appendix A

Glossary of Notations

C: Free choice regarding associated nesting-hierarchy level.

Decisional control (DC): The degree to which decision making has the power to reduce the threat or stress in one's circumstances.

DM: Decision maker.

E(t): Mathematical expectation of threat upon available DC implementation. The probability that a threat will be realized [Q2]after ideal decision making has been done.

N: No choice. External object assignment whose identity is known to the DM at the decisional process outset.

Nesting hierarchy (also decision hierarchy): Stratified organization of choice objects, where objects at subordinate levels (e.g., hospital wards) are nested within objects at superordinate levels (e.g., hospitals).

OSS: Outcome set size, quantifying prevailing DC's *information-processing load*, which refers to the number of predictive judgements required to ascertain the stressor situation's potentially encountered element of least $t_{i.}$

P: Number of bin-nesting bin sets.

p: Number of bin-set nested, element-nesting bins.

Pr(t_1): An index of DC-conveyed threat reducibility comprising the probability of access to a stressor situation's least-threat element.

q: Number of bin-nested elements.

RSS: Response set size, quantifying a situation's decisional controllability.

t_i: Threat value of stressor-situation element i. *Threat* is the likelihood of a given adverse event.

U: External assignment of an object whose identity is unknown to the DM during the decisional process.

Appendix B

Summary expressions for expected threat for clusters of scenario structures ordered on increasing simulation values

Table A1. First-order scenario structures

Scenario structures with unique or identical values of mean E(t)[a]	Summary expressions for E(t)
CC	t_1
CN	E(min $p\ t_i$)[b]
UC	E(min $q\ t_i$)
NC	E(min $q\ t_i$)
CU	$1/qt_1 + (q-1)/q\,\bar{t}_{i'}$[c]
UU	$1/(pq)t_1 + (1 - 1/(pq))\bar{t}_{i'}$
UN	$1/(pq)t_1 + (1 - 1/(pq))\bar{t}_{i'}$
NU	$1/(pq)t_1 + (1 - 1/(pq))\bar{t}_{i'}$
NN	$1/(pq)t_1 + (1 - 1/(pq))\bar{t}_{i'}$

[a] See Table 3.
[b] E(min $x\ t_i$) is the expected value of the minimum of a random sample of x outcomes t_i.
[c] The term $\bar{t}_{i'}$ denotes the mean of $pq - 1$ values of $t_{i'}$.

Table A2. Second-order scenario structures

Scenario structures with unique or identical values of mean E(t)[a]	Summary expressions for E(t)
CCC	t_1
CCN	$\mathrm{E}(\min Pp\, t_i)$[b]
CNC	$\mathrm{E}(\min Pq\, t_i)$
UCC	$\mathrm{E}(\min pq\, t_i)$
NCC	$\mathrm{E}(\min pq\, t_i)$
CUC	$1/pt_1 + (p-1)/p\mathrm{E}(\min q\, t_{i'})$[c]
CNN	$\mathrm{E}(\min P\, t_i)$
UCN	$\mathrm{E}(\min p\, t_i)$
UUC	$\mathrm{E}(\min q\, t_i)$
UNC	$\mathrm{E}(\min q\, t_i)$
NCN	$\mathrm{E}(\min p\, t_i)$
NUC	$\mathrm{E}(\min q\, t_i)$
NNC	$\mathrm{E}(\min q\, t_i)$
CCU	$1/qt_1 + (q-1)/q\bar{t}_{i'}$[d]
CUN	$1/(pq)(t_1 + (p-1)\bar{t}_{i'}) + (q-1)/q\mathrm{E}(t_{i'} \ni t_{i'} \cap \min Pp\, t_{i'} \in S,$ *where size of S is p* $)$[e]
CNU	$1/(pq)(t_1 + (q-1)\bar{t}_{i'}) + (p-1)/p\mathrm{E}(t_{i'} \ni t_{i'} \cap \min Pq\, t_{i'} \in S,$ *where size of S is q* $)$
UCU	$1/(Pq)(t_1 + (q-1)\bar{t}_{i'}) + (P-1)/P\mathrm{E}(t_{i'} \ni t_{i'} \cap \min pq\, t_{i'} \in S,$ *where size of S is q* $)$
NCU	$1/(Pq)(t_1 + (q-1)\bar{t}_{i'}) + (P-1)/P\mathrm{E}(t_{i'} \ni t_{i'} \cap \min pq\, t_{i'} \in S,$ *where size of S is q* $)$
CUU	$1/(pq)t_1 + (1 - 1/(pq))\bar{t}_{i'}$
UUU	$1/(Ppq)t_1 + (1 - 1/(Ppq))\bar{t}_{i'}$
UUN	$1/(Ppq)t_1 + (1 - 1/(Ppq))\bar{t}_{i'}$
UNU	$1/(Ppq)t_1 + (1 - 1/(Ppq))\bar{t}_{i'}$
UNN	$1/(Ppq)t_1 + (1 - 1/(Ppq))\bar{t}_{i'}$
NUU	$1/(Ppq)t_1 + (1 - 1/(Ppq))\bar{t}_{i'}$
NUN	$1/(Ppq)t_1 + (1 - 1/(Ppq))\bar{t}_{i'}$
NNU	$1/(Ppq)t_1 + (1 - 1/(Ppq))\bar{t}_{i'}$
NNN	$1/(Ppq)t_1 + (1 - 1/(Ppq))\bar{t}_{i'}$

[a] See Table 4.
[b] $\mathrm{E}(\min x\, t_i)$ is the expected value of the minimum of a random sample of x t_i outcomes.
[c] $\mathrm{E}(\min x\, t_{i'})$ is the expected value of the minimum of a random sample of $x\, t_{i'}$ outcomes.
[d] The term $\bar{t}_{i'}$ denotes the mean of $Ppq - 1$ values of $t_{i'}$.
[e] Elaboration upon terms in this set of expressions is available according to the latter's full computational formulae, as presented for CNU in Table 2, and for NCU in equation (4) and its related prose, of the text.