Online Supplementary Material

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1) Narrative for E(t) for first-order scenario NC, including the operation of the Hypergeometric Distribution (Patil & Joshi, 1968).

For Scenario NC,

$$E(t) = \frac{1}{p}t_1 + \frac{p-1}{p} \cdot \frac{q}{pq-1} \sum_{i'=2}^{pq} H(q-1;\ pq-2, pq-i', q-1) \cdot t_{i'}$$

Expected threat $E(t)$ for NC comprises the sum of two expressions. The first again is simply the product of $Pr(t_1)$ and (t_1); that is, the bin containing t_1 is assigned with probability $1/p$, and if assigned, t_1 is chosen. The first term of the second expression, $(p-1)/p$, represents the complement of $Pr(t_1)$, the probability that a t_i other than t_1 will be selected. The second fraction $q/(pq-1)$ represents the probability that for any particular t_i, the latter is located in a bin other than that of t_1, that is $(p-1)q/(pq-1)$, multiplied against the probability that the bin containing $t_{i'}$ is assigned given that the one containing t_1 has not been assigned, or $1/(p-1)$. Each of the summed terms in the second expression involves the hypergeometric distribution (e.g., Patil & Joshi, 1968), used to obtain the probability that $t_{i'}$ is the lowest threat value in its bin, given that the bin with t_1 is not assigned and that $t_{i'}$ is not in t_1's bin. The Hypergeometric distribution is used to assess the probability of a given $t_{i'}$ thus being selected, which is tantamount to the probability that all other elements in its current bin have higher threat values. Paralleling bin-model terminology, H(q-1; pq-2, pq-i', q-1) is the probability that out of a random sampling of q-1 (mutually independent) balls, without replacement (cf., Milenkovic, 2004) -- that is, q-1 (mutually independent) threat values $t_{i'}$ -- from a bin containing a total of pq-2 balls (both t_1 and the $t_{i'}$ under consideration themselves are ineligible), pq-i' of which are white, -- that is, pq-i' for which $t_{i'}$ exceeds the particular $t_{i'}$ under consideration, -- all q-1 sampled balls are white -- that is, all sampled $t_{i'}$ values exceed the currently entertained $t_{i'}$. An analogous format of the hypergeometric distribution is used when the latter is called upon in obtaining the remaining mathematical expectations of threat.

Note that H(q-1; pq-2, pq-i', q-1) is equal to $1-\sum_{j=0}^{q-2} H(j;\ pq-2, pq-i', q-1)$.

Moreover, obtaining q-1 white balls in a random sampling of size q-1 implies obtaining 0 black balls (i.e., obtaining no $t_{i'}$ values lower than the considered $t_{i'}$), with the equivalent probability now expressed as $H(0;\ pq-2,\ i'-2,\ q-1) = 1-\sum_{j=1}^{q-1} H(j;\ pq-2, i'-2, q-1)$.

Along with the pq element encounters being mutually exclusive and exhaustive, whereby constituent probabilities involved in $E(t)$ sum to 1.0, these relations are available as computational checks regarding $E(t)$ for structure NC, and by extensions prescribed by their host scenario structures, all other $E(t)$ formulae.

2) Attribute values and narrative formulation of E(t) for second-order scenario UCN

Scenario UCN.

$Pr(t_1) = 1/Pq$

$RSS = p$;

$$E(t) = \frac{1}{P}\left(\frac{1}{q}t_1 + \frac{q-1}{q}\cdot\frac{p}{Ppq-1}\sum_{i'=2}^{Ppq} H(p-1;\ Ppq-2, Ppq-i', p-1)\cdot t_{i'}\right)$$

$$+\frac{P-1}{P}\cdot\frac{(P-1)p}{Ppq-1}\cdot\frac{1}{P-1}\sum_{i'=2}^{Ppq} H(p-1;\ Ppq-2, Ppq-i', p-1)\cdot t_{i'}$$

$OSS = Pp.$

The value of $Pr(t_1)$ corresponds to the inverse of the product of the set sizes (P and q) of the hierarchy levels at which free choice is not exercised (U and N). The *RSS* corresponds to the set size (p) of the tier with free choice. The *OSS* is the product of the two set sizes of the tiers at which there are multiple possibilities (P and p under U and C, respectively).

There are two larger expressions in the UCN $E(t)$ formula. The first fraction applied to the entire first expression is $1/P$, the probability that t_1's bin set is selected. Within the large bracket, the first term $1/q$, as combined with $1/P$, accounts for $Pr(t_1)$, which is scaled by t_1. The first fraction in the second term within the bracket represents the complement of $1/q$, the probability that t_1 is not assigned ($(q-1)/q$). The second fraction $p/(Ppq-1)$ represents the probability that $t_{i'}$ is one of those assigned into t_1's bin set, given that t_1 itself is not assigned. The appearance of the hypergeometric distribution within this first larger expression conveys the probability that $t_{i'}$ is the least of the $t_{i'}$'s assigned in t_1's bin set, given that t_1 itself is not assigned. The hypergeometric-distribution probabilities multiplying respective $t_{i'}$ values are summed over i' values.

The second larger expression starts with the probability that t_1's bin set is not assigned((P-1)/P). The next fraction ((P-1)p/(Ppq-1)) represents the probability that $t_{i'}$ is one of the assigned $t_{i'}$s located in bin sets other than t_1's bin set. The fraction 1/(P-1) then accounts for the probability that the bin set containing $t_{i'}$ is assigned, given that t_1's bin set has not been assigned. The second use of the hypergeometric distribution yields the probability that $t_{i'}$ is the least of the p $t_{i'}$s assigned in $t_{i'}$'s bin set given that t_1's bin set is not assigned. This second larger expression then entails the summation of these probabilities of $t_{i'}$ encounters combined with the respective $t_{i'}$ values.

3) Threat Expectation E(t) and Unpredictability of Threatened Events

Implementation of decisional control conveys specific values of threat expectation $E(t)$ according to a scenario structure's prevailing choice conditions. Decisional-control determined $E(t)$ also enters into adverse-event predictability, as follows. Level of threat identified with scenario-element i, defined as the probability of adverse-event occurrence t_i, does not directly stipulate the impact, or magnitude, of the stochastic event itself (cf., Neufeld, 1990; Paterson & Neufeld, 1987). However, implicit in the current expression of structure-wise threat level $E(t)$ are two event magnitudes (m), specifically 1 "unit of severity", corresponding to event occurrence, and 0 units, corresponding to event non-occurrence:

$$\sum_{i=1}^{(P)pq} Pr(t_i)[t_i(1.0)+(1-t_i)(0)] = \sum_{i=1}^{(P)pq} Pr(t_i)t_i = E(t). \qquad (5)$$

This dichotomous format of event magnitude nevertheless is coherent with associational-memory ("categorical memory") accounts of probability learning (Estes, 1975; 1976; 1977), which repeatedly have been shown to extend to predictive judgments of stressor-event occurrence (Morrison, et al, 1988; Mothersill & Neufeld, 1985; Neufeld & Herzog, 1983; Lees & Neufeld, 1999). Greater variation in event magnitude m, however, can be accommodated in the present $E(t)$ computations; where the adverse event corresponding to scenario-element i is of an unique magnitude m_i, for example, its cross-product with the event's probability of occurrence t_{m_i}, that is $t_{m_i}\ m_i$, can be stipulated to equal the element's value of t_i in the present layout. $Var(m)$, in turn becomes

$$\sum_{i=1}^{(P)pq} Pr(t_{m_i})\ t_{m_i}\ m_i^2 - [\sum_{i=1}^{(P)pq} Pr(t_{m_i})\ t_{m_i}\ m_i\ ]^2 .$$

Exercise of available control thus affects level of stressor-event threat, as expressed in $E(t)$; however, it also impinges on stressor-event predictability in quantifiable ways directly incorporating $E(t)$. Note that stress activation is deemed to be driven upward as predictability of stressor characteristics, including event magnitude, declines (see, e.g., Denuit & Genest, 2001; Osuna, 1985; Paterson & Neufeld, 1987; Smith, 1989; Suck & Holling, 1997). Accordingly, variance in event magnitude $Var(m)$ can be shown to equal $E(t)[1\text{-}E(t)]$. Formally,

$$m \in \{0,1\};\ t_i \in [0,1];$$

$$Var(m) = E[E(m^2 \mid i)] - [E(E(m \mid i)]^2$$

$$= \sum_{i=1}^{(P)pq} Pr(t_i)\ [t_i\ (1^2) + (1-t_i)\ (0^2)] - \{\sum_{i=1}^{(P)pq} Pr\ (t_i)\ [t_i\ (1.0) + (1-t_i)\ (0)]\}^2$$

$$= \sum_{i=1}^{(P)pq} Pr(t_i)\ (t_i) - [\sum_{i=1}^{(P)pq} Pr(t_i)\ t_i]^2 = E(t)\ [1-E(t)]. \qquad (6)$$

Consequently, $Var(m)$ is maximized where $E(t) = 0.5$. This value turns out to be approximated by the larger values of $E(t)$ obtained in the simulation results presented in Tables 3 and 4 (see main document; further considerations surrounding $Var(m)$ are available as a .pdf document from the first author.) In this way, those scenario structures with elevated threat $E(t)$ also are accompanied by elevated unpredictability $E(t)[1\text{-}E(t)]$.

Other indexes of (un)predictability, framed within the present quantitative structure, convey additional situation properties that stand to be psychological-stress significant. One such index, which ignores event magnitude, is $Var(t_i)$,

$$\sum_{i=1}^{(P)pq} Pr(t_i)\ t_i^{\ 2} - [\sum_{i=1}^{(P)pq} Pr(t_i)\ t_i\ ]^2 .$$

Another, which circumvents event magnitude, is Shannon-Weaver information entropy:

$$-\sum_{i=1}^{(P)pq} Pr(t_i)\log_2[Pr(t_i)] =$$

$$-\sum_{i=1}^{(P)pq} Pr(t_{m_i}\, m_i)\log_2[Pr(t_{m_i}\, m_i)];$$

$$Pr(t_i) = Pr(t_{m_i}\, m_i).$$

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