A Case For Chaos Theory Inclusion In Neuropsychoanalytic Modelling

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This paper provides a review of the current evidence of chaoticity at various scales of the brain-mind as well as the application of nonlinear tools in clinical practice. Based on these data, a hypothesis is formulated that the brain-mind at various scales can operate in linear, nonlinear, or hybrid modes, such as chaotic functioning accompanied by noise. A thesis formulated by Mark Solms (2021) that living systems must minimize Shannon's entropy of the physical states (sensory entropy) is considered. Based on the data presented in this paper minimization of entropy in that sense appears to be describing only a part of the complex brain-mind dynamics. Studies evaluating measures of entropy specifically developed for real living systems such as discrete timescale entropy (ApEn) suggest that a decrease in EEG entropy can be observed in some neuronal processes (e.g. progression from wakefulness to deep sleep); however, EEG entropy is observed to be increasing at other times and in other modes of brain-mind functioning (e.g. progression from deep sleep to REM to wakefulness; and from vegetative state to wakefulness). The clinical implications are discussed. This paper proposes that it would be theoretically and clinically beneficial for future revisions of the neuropsychoanalytic models to consider including the chaos theory framework.

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Introduction

Karl Friston's paper "Life as we know it" (2013) presented a stochastic model and a computer simulation of self-organization arising from a "primordial soup" under specific assumptions. Attempting to draw connections from his theory to other models, Friston stated the following: "Finally, because we have cast this treatment in terms of random dynamical systems, there is an easy connection to dynamical formulations that predominate the neurosciences [40, 45-47]" (p.5). One of the dynamical formulations cited by Friston was a paper by Walter Freeman (1994), a pioneer of modelling neuronal processes with the chaos theory framework. It seems that Friston saw that his model could possibly be extended to include chaotic dynamical systems at some point in the future.

Mark Solms (2020a) incorporated Friston's mathematical model as a component in his seminal "New Project for a Scientific Psychology: General Scheme." In a commentary on this paper, Robert Galatzer-Levy (2020) expressed a view that the possibility of the brain-mind functioning at times as a non-linear system was not sufficiently considered. In response, Solms (2020b) suggested that the key point in such discussion was whether the mental apparatus was indeed non-linear (p. 102).

An attempt to explore the answer to this question is the focus of this paper[1].

Scaling

Brain-mind phenomena exist at various levels of spatial and temporal scales from a synapse to the entire brain and from milliseconds to multiple hours. Within the same spatial or temporal scale, it is useful to identify the appropriate mathematical tools and the level of model complexity that are necessary to describe the phenomena studied. Stochastic differential equations might be appropriate at one scale and nonlinear time

series analysis (NTSA) of EEG at another. Modelling the dynamics of a single isolated neuron might require a less complex model than modelling a network of interconnected neurons – due to emergent properties in a network that are not present in isolated neurons.

As we travel up or down the scale in the brain-mind, we can observe changes in some parameters and invariance in others. If, for example, we observe self-similarity in the branching pattern when traveling up or down the tree of axons in a single neuron, we could suspect a scale invariant aspect. Should we see the evidence of a neuronal system being both linear and deterministic in nature, we could apply a reduction in space or time as a method of studying this system. If we have evidence of the system being chaotic or stochastic, then reduction would not be possible, as such systems cannot be reduced.

Key concepts

A series "Is there chaos in the brain" was published by Faure and Korn in two parts. The first part (2001) introduced the concepts of nonlinear dynamics and methods of investigation. The second part (2003) discussed the experimental evidence. Since Faure and Korn covered the history of determinism and chaos extensively, I will not repeat it here. Most of the terms used in this paper are defined in the Glossary section of Daniel Toker's paper "A simple method for detecting chaos in nature" (2020)[2]. In what follows, I will only highlight some key differences in the types of dynamical systems.

Dynamical systems can be broadly characterized as linear or nonlinear, each of which can be stochastic or deterministic (Toker, 2020)[3].

Linear deterministic systems are completely predictable, both short-term and long-term. An example would be an object moving with fixed velocity on a straight line. Stochastic systems are unpredictable short-term and long-term due to the presence of

random components. Examples of stochastic systems include a coin toss or mitochondria segregation during cell division.

Chaotic systems are somewhat predictable short-term but are not predictable long-term due to sensitive dependence on initial conditions (SDIC), which is commonly called the "butterfly effect." An example of a chaotic system is weather – one can reasonably accurately provide a three-day temperature forecast in Rome, Italy, but not an eight-month forecast.

Chaotic systems are nonlinear and forward deterministic (Frigg 2020), which means that should the starting state X(0) be known precisely, the equations governing the chaotic system would allow us to calculate the system's mature state X(t). With that, there is nearly always incomplete information, measurement error, or noise in assessing the starting state for real-world phenomena and the nature of chaotic systems is such that even minor differences in the starting state would lead to significant differences in mature states due to exponential rate of divergence of initially nearby trajectories[4]. Therefore, real-life chaotic systems are not predictable long-term.

In addition to the "pure," theoretical linearity, stochasticity, and chaos, real living systems can have multiple components present concurrently, such as a primarily chaotic system with some noise (random factor)[5].

In what follows, I will appeal to measures of entropy and complexity to see which principles can be brought to bear on sentient behavior. Before doing this, it will be useful to note that terms like entropy and complexity are typically attributes of probability distributions. This means one has to be explicit about which probability distribution the entropy pertains to. The free energy principle — upon which Mark Solms' formulation (2020a) rests — deals with minimizing the entropy of physical states (e.g., sensations) induced by exchange with the world. This means that to exist in

some characteristic states, is simply to place an upper bound on the entropy or dispersion of sensory exchanges with the environment. Because the path integral of variational free energy is an upper bound on this entropy, simply existing in some characteristic states can be described as minimizing sensory entropy.

In statistics, these quantities are also known as negative log Bayesian model evidence. On this view, minimizing sensory entropy is equivalent to maximizing the evidence for an agent's model of her world; usually referred to as a generative model that generates predictions of sensations from inferred causes. Interestingly, this self-evidencing (Hohwy, 2016) paradoxically maximizes the entropy of Bayesian or representational beliefs about those causes, in accord with Jaynes' maximum entropy principle. Under the free energy principle, these (Bayesian) beliefs are thought to be encoded by physical neuronal states that have a low entropy – because the sensory perturbations that cause neuronal dynamics have a low entropy. When Mark Solms talks about the minimization of entropy (2020a) it is the entropy of physical (sensory or neuronal) states that is minimized. But what does this mean for the complexity of neuronal dynamics?

We now face the delicate issue of moving from probability distributions over states to measures of their dynamics. A useful picture here is a set of trajectories through state space that has certain properties[6]. In brief, the density of trajectories in any regime of state space can be read as the probability of finding the system in that regime of state space, when sampled at random. This means that the entropy of this probability density measures the volume of a neuronal state space that is characteristically occupied over a suitably long period of time. This volume corresponds to the attracting set of states that are characteristic of the system; referred to as a manifold or attractor. In short, entropy is the volume of the characteristic or attracting set.

However, this does not reflect or quantify the complexity of the dynamics. In other words, the attracting set can be 'space filling' but have a low volume. This is a characteristic feature of chaotic dynamics that show an exponential divergence of trajectories. This necessarily involves itinerant and unpredictable dynamics that may be very complex and yet have a low entropy. Examples of these itinerant dynamics are the chaotic oscillations at multiple scales seen in the brain, ranging to fast gamma through to ultra-slow fluctuations and, indeed, the sleep-wake cycle. So why is this important?

The thesis pursued in this monograph is that simply minimizing (or upper bounding) the entropy or volume of characteristic neuronal states is not a complete description of a sentient brain. One also has to pay attention to the complexity entailed by the chaotic itinerancy and delicately structured manifolds on which neuronal trajectories unfold. This complexity itself can be subject to chaotic itinerancy, for example, the differences in dynamics during wakefulness, various sleep stages and pathological conditions. In what follows, we will look at both the theoretical and empirical implications of this more complete formulation of the neuronal correlates of sentience.

Measures

Two variables are typically used to assess the chaoticity of ideal systems: Largest Lyapunov Exponent (LLE) and Kolmogorov-Sinai Entropy (K-S entropy) (Faure & Korn, 2001). Positive LLE or positive K-S entropy can be used to classify the system as chaotic. In addition, Correlation Dimension (CD) is used to assess the system's complexity (Faure & Korn, 2001).

Since Mark Solms refers to Shannon's entropy frequently in his book (2021) and paper (2020a), it seems important to try drawing some connections and highlighting

differences between Shannon's and K-S entropy. Shannon's entropy (1948) is a common measure of uncertainty of a future outcome, or, more specifically, it can be described as a measure of average receiver's uncertainty about what message the source produces next.

As Frigg (2004) points out, the conceptual frameworks and the mathematical tools used by Shannon (1948) and dynamical systems theorists are different – "the former deals with a finite set of discrete messages and their combinations, while the latter considers a continuous measurable phase space on which an automorphism (function mapping the phase space onto itself) is defined" (p. 4). For this and other reasons listed by Frigg, it is not easy to answer how to relate entropies in the communication theory and topological theories. However, Frigg's paper is an attempt to provide some conceptual connection between the two entropies despite such challenges. He has shown that K-S entropy can be seen as an analogous measure to a *generalized version* of Shannon's entropy under certain plausible assumptions; therefore, one can also consider K-S entropy as a measure of uncertainty of future events when past events are known.

Similarly to the assessment of ergodicity or stochasticity, assessing chaoticity in real living systems is significantly more challenging than in ideal ones. The abovementioned classical measures of chaoticity are highly sensitive to noise and are challenging to calculate for short time series (Toker, 2020). To address these challenges, the researchers investigating possible chaoticity in real living systems typically introduce additional measures and controls in their methodology.

Toker (2020) used Permutation Entropy (PeEn), which in discrete time series is equivalent to K-S entropy. He additionally developed a multi-layered process, called Chaos Decision Tree Algorithm, which allowed not only to assess systems for stochasticity or chaoticity but also to reduce noise and correct for oversampling. Several studies based on nonlinear time series analysis of EEG data used Approximate Entropy

(ApEn), which approximates K-S entropy and was developed by Pincus (1991) for short and noisy data sets. A surrogate data method (Kantz & Schreiber, 2003) was used by researchers who studied nonlinear time series analysis to distinguish the effects of noise from the effects of several nonlinear elements interacting with each other (Korn & Faure, 2003). Adeli and colleagues (2007) combined classical chaoticity measurements with wavelet methodology.

Such variability in methods and measures can be off-putting to some readers, however, it is not unique to chaoticity assessment in real-world living systems but equally applies to all other types of dynamical systems, such as stochastic ones. Importantly, with the "checks and balances" of statistical controls in place, the core idea of chaoticity assessment remains the same – the system is analyzed for the exponential divergence of trajectories in phase space (Toker, 2020), which, if detected, indicates chaoticity.

Assumptions and tools in the current neuropsychoanalytic theories

Friston's model of self-organization based on Markov blankets (2013) relies on an assumption of the system's ergodicity. While ergodicity is frequently assumed in theoretical systems, the formal proof of ergodicity for a theoretical or real-life system is a rare occurrence. Indeed, while the concept of ergodicity was introduced in the 1870s by Ludwig Bolzman (Ashley, 2015), the first formal proof of ergodicity for a theoretical system – Sinai Billiards – was completed by Yakov Sinai in 1963 (Ashley, 2015). Unlike Sinai's ideal system, the question of proof of ergodicity of any real living system is far from certain.

An assumption of ergodicity may apply to some isolated components of living systems, but not to others. For example, at a micro-level, Weigel et al., (2011) suggested the coexistence of both ergodic and nonergodic processes in the plasma membrane of a

single cell; Weron et al., (2011) described conditions when ergodicity broke down on the surface of hippocampal neurons. On a macro level, Medaglia et al., (2011) suggested that in neural network modelling ergodicity was often assumed, while data were not always tested for ergodicity, leading to issues with the interpretation of results.

Friston (2013) wrote about the assumption of ergodicity as being a clear simplification for real biological systems (p.11). In an earlier paper, he (2010) made a more specific statement about the limitations of such an assumption:

> Clearly, the ergodic assumption in S1.1 only holds over certain temporal scales for real organisms that are on a trajectory from birth to death. This scale can be somatic (e.g., over days or months, where development is locally stationary) or evolutionary (e.g., over generations, where evolution is locally stationary). (p.140)

Therefore, when we apply Friston's model to the brain-mind at a specific spatial and temporal scale, it would seem preferable to test the system for ergodicity than to assume it. We just do not know a priori if a particular neuronal system is ergodic in a specific state of the brain-mind.

The basic scientific methods used by Solms and Friston are based upon a variational principle of least action, where action is the path integral of free energy that upper bounds entropy. This means that one can regard self-organization or self-evidencing as reducing the dispersion of sensory states; for example, interoceptive states — leading to homeostasis.

At times, however, my reading of the application of this principle seems to resemble a scientific method of reduction, by which I mean deconstructing the whole into

component parts, describing a phenomenon at the level of the specific components to suggest that this component-level phenomenon significantly contributes to (or sometimes fully explains) the macro phenomena at the level of the whole[7].

For example, Friston (2010) wrote: "indeed, the physiology of biological systems can be reduced almost entirely to a homeostasis [7]" (p. 127). Similarly, Solms (2020a) described sensory neurons as "homeostatic receptors" and motor neurons as "homeostatic effectors" (p.7); Solms (2019) described the elemental form of consciousness as affect, the physiological mechanism of which he considered to be an extended form of homeostasis. Solms' proposed model of perception (2020a) consisted of the nested hierarchy of billions of homeostats and more generally cathected predictive hierarchy (2020a) was composed of billions of homeostats.

If some elements of reduction are used, the key question would be whether reduction can work in principle for a particular kind of system. Linear systems can be reduced in time or space. Classically chaotic (not hybrid) or stochastic systems cannot be reduced in the same way as linear systems in the abovementioned sense of reduction.

The equations in Friston's (2013) paper are stochastic differential equations. The "Heuristic Proof" (p.2) part of the paper is based on random dynamical systems (RDS). One of the consequences of Friston's choice of these tools is his acceptance of randomness as a participant component in the systems he is modelling. Let me consider this, particularly as it applies to the integration of Friston's work in Mark Solms's (2020a) "New Project for a Scientific Psychology: General Scheme." Since Solms' paper is a revision of Freud's original paper (1950) I will first consider Freud's views on possible presence of random phenomena in the mind.

Freud postulated a principle of psychic determinism, which became one of the foundational laws in psychoanalysis. Perhaps, the clearest citation to illustrate it from his

"Psychopathology in Everyday life" (1901) is the following: "But there is nothing arbitrary or undetermined in the psychic life" (p. 345). Further, Freud clarified that motivation for action or thought can be conscious or unconscious, but "the determinism in the psychic realm is thus carried out uninterruptedly" (p.374). I interpret these formulations as Freud's belief of randomness not existing in the mind.

In the Solms' (2020a) revised version of the Project, the author did not seem to explicitly state if Freud's postulate of psychic determinism was preserved. My reading of the section "Dream Content" (p. 27) suggests that determinism is maintained in Solms' description of "unconscious intermediate links, which we can easily discover when we are awake" (p.27).

Should Freud's principle of psychic determinism be preserved in Solms' "New Project for a Scientific Psychology: General Scheme," could we possibly run into an issue of two mutually exclusive assumptions present in the same model? Mark Solms (2020a) has built his theory of the mental apparatus while integrating Friston's stochastic model of self-organization (2013) in the section of the New Project called "The functioning of the apparatus" (p.14). Consistent with the dual aspect monism perspective, the mental apparatus is an abstraction that is seen as the brain when observed from a position external to the body and as a mind when observed subjectively, from within the body (Solms, 2018). Solms (2020a) incorporated Friston's model to describe both subjective (consciousness) and objective (neuronal) perspectives. Therefore, it is reasonable to conclude that mental apparatus is isomorphic with Friston's model, which necessitates the presence of randomness in it. However, Freud's principle of psychic determinism implies the absence of randomness in the mind. Perhaps, there is a possibility to clarify Solms' position on the presence of psychic determinism as an assumption in his "New Project for a Scientific Psychology: General Scheme."

The pushback against this argument is that although the dynamics of the states may be stochastic the density dynamics are deterministic. In other words, the evolution of the probability density over the states of a stochastic (and possibly chaotic) system is deterministic. For example, the variance of a random variable is not a random variable – it is a sufficient statistic. Put simply, this means that if neuronal activity encodes the sufficient statistics of probabilistic beliefs about random stochastic states of the world, then neuronal dynamics and their representations are deterministic in some sense. However, the things that they represent can be stochastic – and their representation can show deterministic chaos.

In some sense, this is the whole point of the free energy principle, in which internal brain states are read as encoding or representing beliefs about random variables that cannot be directly observed in the outside world. This belief-based approach to representationalism is often dominated by the representation of uncertainty that figures implicitly in many Freudian formulations.

Another consequence of Friston's (2013) choice of mathematical tools is the use of discrete variables ("Proof of Principle" section, p.5). When we apply Friston's model to phenomena at different scales in the brain-mind, might it be useful to consider the possible influence of using continuous or discrete variables for the studied objects? Solms (2020a) seemed to use discrete variables in the section "The problem of quantity" (p.9) and referred to continuous ones in the "Consciousness" section (p.14). Solms (2020a) referenced Shannon's entropy in his paper, which was defined by Shannon as a discrete variable[8].

Various techniques can be used to approximate continuous phenomena with discrete data, the most common being sampling. The process of sampling is complex and in and of itself can lead to significant distortions.

In the addition to the sampling distortions, on a conceptual level, when we model continuous phenomena with discrete variables and discrete phenomena with continuous variables, we may run into misrepresentations of data. Robert Sapolsky (Stanford, 2011) provides an example of using categories "warm" and "hot" to describe water temperature. This approach would create an impression of an important boundary between warm and hot, which does not exist in a continuously measured temperature.

It seems that in Mark Solms' book "The hidden spring: a journey to the source of consciousness" (2021), the choice to use discrete variables at times led to conclusions that seem to differ from those that could be achieved with continuous variable modelling. Consider the following text:

Most people have an intuitive understanding of what "entropy" is. They think of it as a natural tendency towards disorder, dissipation, dissolution, and the like. The laws of entropy are what make ice melt, batteries lose their charge, billiard balls come to a halt and hot water merge with cold.

Homeostasis runs in the opposite direction. It resists entropy. It ensures that you occupy a limited range of states. That is how it maintains your required temperature, and how it keeps you alive – how it prevents you from dissipating. Living things must resist one of the fundamental principles of physics: the Second Law of Thermodynamics. (p. 154)

Here, the "limited range of states" mentioned along with an example of maintaining [body] temperature, seems to assume that you measure temperature discretely, and perhaps, in integer values, then indeed a range from 36 to 38 degrees

Celsius would be limited. However, when temperature is a continuous variable, there is an infinite number of temperatures between 36 and 38 degrees. Consequently, it would be possible for a chaotic system to operate within this seemingly narrow range and for the K-S entropy of this system to have a stable positive value. An example would be a Lorenz system (1963), which exhibits chaotic behavior. Strange attractor in such a Lorenz system occupies limited volume in phase space; however, it does not occupy a "limited range of states," quite the opposite – it never crosses the same point in phase space.

Of course, there may be no bright line between continuous and discrete representations. Indeed, it is not uncommon to suppose that the brain might use both at the same time; namely, continuous state space models to process a continuous time engagement with the world, that is modelled in terms of discrete representations that are more apt to handle things like concepts and language (Friston et al., 2017).

Another aspect - the second law of thermodynamics mentioned in Solms' (2021) book - was formulated for isolated systems (Planck, 1926). Living things exchange matter with the outside world. Second law of thermodynamics does not apply to open or semi-open systems. Karl Friston (2013) noted this difference in his paper: "This enables biological systems to resist the second law of thermodynamics—or more exactly the fluctuation theorem that applies to open systems far from equilibrium" (p.2). Here, he addressed not only the isolation condition, but also the second issue – equilibrium. Thermodynamic properties, such as thermodynamic entropy, were defined for the states of equilibrium. Living things do not meet this criterion.

Finally, Friston's (2013) simulation was based on a collection of 128 subsystems (p.6) that were identical to each other at the start, which is a homogenous set. This homogeneity stands in contrast to the diversity of cells in the human brain – Tasik and colleagues (2018) showed evidence of 133 different types of cells in just the neocortex.

Friston was aware of homogeneity being a simplification: "An interesting challenge now will be to simulate the emergence of multicellular structures using more realistic models with a greater (and empirically grounded) heterogeneity and formal structure" (p.11).

To summarize, stochasticity, ergodicity, a possible use of reduction and discrete variables, and homogeneity are some of the assumptions and tools in Friston's (2013) model of self-organization and Solms' (2020a) model of the mental apparatus and its functioning. Might the application of these models to real living systems, such as the brain-mind, possibly benefit from an evaluation of these systems for stochasticity, chaoticity, linearity, and homogeneity – at each level of the scale from a single neuron to consciousness? Toker (2020) proposed a practical and reasonably accurate method to evaluate and classify real live systems as stochastic, chaotic, or periodic.

Evidence of chaoticity in the brain-mind

Animal research literature demonstrating the presence of chaotic processes at the brain scales ranging from isolated squid axons to single neurons, and to coupled neurons is reviewed in Korn and Faure (2003).

In research with human subjects, Y. Ma et. al (2018) reviewed the studies of nonlinear analysis of adult EEGs during different stages of sleep, including 13 studies evaluating entropy (e.g. ApEn) and 21 studies evaluating complexity with fractal-based methods (e.g. CD). A general trend in these studies showed both entropy (ApEn) and complexity growing in the same progression from the deepest N3 stage of sleep (lowest entropy) to N2, N1, REM, and Wakefulness (highest entropy). Lee et. al. (2013) showed the same progression for children, and that for each stage of sleep entropy values were lower for children than adults.

Similar pattern was reported by Mateos and colleagues (2016) who calculated entropy (PeEn) based on recording of scalp EEG in some subjects and intracranial EEG in others during various phases of sleep and resting wakefulness.

Another EEG study by Liley and colleagues (2010) was focused on the alpha range (8-13Hz). They presented their own model of the alpha rhythm and provided some experimental evidence. Specifically, a parieto-occipital recorded EEG of a healthy male subject showed a detection of weak non-linearity. Based on the empirical data and their theory, Liley and colleagues formulated a hypothesis that alpha was a "readiness" rhythm, suggesting "cortex perched on the brink of stability, which when perturbed gives rise to a range of unanticipated complex dynamics that include 40 Hz (gamma) activity" (p.1). My reading of their hypothesis is that they saw alpha as a phase transition – an onset of bifurcations, leading to system changing into a chaotic mode. Further research is necessary to test this hypothesis.

Evidence of chaoticity and use of chaos theory tools in clinical applications[9]

Sarà & Pistoia (2010) studied EEG recordings of ten patients in a Vegetative State (VS). Entropy (ApEn) of the EEG time series was calculated and compared to that of ten heathy controls. The authors showed that mean ApEn values were lower in patients than controls, supporting their hypothesis of decreased complexity in VS.

Thul and colleagues (2016) analyzed EEG of 15 severely-brain-damaged subjects and compared their EEG recordings of those of 24 healthy controls. They have showed that entropy (PeEn) was lowest in patients in VS; it was higher in Minimally Conscious State patients and highest in healthy controls.

Mateos and colleagues (2016) studied different states of consciousness and alertness in healthy subjects and patients with epileptic seizures (27 subjects total), using scalp EEG, intracranial EEG (iEEG), and magnetoencephalography (MEG). The authors

subsequently assessed the data for entropy and complexity, using various measures, including PeEn. Based on all three kinds of recordings, they showed that "the values of entropy and complexity of the signals tend to be greatest when the subjects are in fully alert states, falling in states with loss of awareness or consciousness" (p.73). In terms of alertness, they showed a noticeable decrease in complexity and entropy when awake subjects closed their eyes.

Adeli and colleagues (2007) studied EEGs of healthy subjects, patients with epilepsy between seizures and during seizures. In each group they used 100 single channel EEG recordings; they used wavelet processing (Adeli & Ghosh-Dastidar, 2010) of specific EEG subbands combined with chaoticity assessment. Specifically, they assessed attractor chaoticity with LLE and attractor complexity with CD. Their results showed highest chaoticity (LLE) in a healthy subject group for band-limited EEG (0-60Hz). They also showed lowest complexity (CD) for the seizure group for band-limited EEG (0-60Hz).

These results obtained by Adeli and his colleagues suggested lower K-S entropy during seizure as compared to non-seizure, supporting the conclusion reached by Mateo et al. (2016). Adeli and Ghosh-Dastidar published a book in 2010, called "Automated EEG-based diagnosis of neurological disorders: Inventing the future of neurology," in which they described an automated methodology for epilepsy diagnosis and seizure detection that they have developed, which, they claimed, reached an accuracy of 96%. I have not found independent verifications or follow-up studies of this claim.

Darbin et al. (2013) used non-linear technique of EEG analysis (ApEn) as part of the model they developed to assess parkinsonism. They reviewed parkinsonian primate research data, where deep brain stimulation (DBS) of the subthalamic nucleus (STN) led to changes in the nonlinear features in the globus pallidus internus (GPi) neuronal stream

(Dorval, et al., 2008). Darbin and colleagues confirmed this finding in human Parkinsonian patients when they observed a decrease in neuronal entropy in the STN during the DBS accompanied by the administration of dopaminergic agonist apomorphine (Lafreniere-Roula et al., 2010). They suggested that higher entropy (ApEn) in the globus pallidus internus GPi and subthalamic nucleus STN was associated with hypokinetic conditions.

More recently, Darbin and colleagues (2020) used machine learning algorithms and non-linear framework while analyzing three groups of primates – healthy, parkinsonian off-medication, and parkinsonian on-medication. The machine learning algorithms achieved nearly 0.90 accuracy in discriminating between these conditions.

Related in some aspects of modelling approach to the work of Darbin and colleagues and going back to epilepsy, Müller et al. (2020) studied patients with drug resistant epilepsy who underwent surgery. The authors used network-based approach to epilepsy modelling and found the non-linear interrelations between the iEEG signals to be a marker of epileptogenic tissue, which can possibly be used in the future to help improve the accuracy of selecting tissues for resection.

Zolezzi et. al. (2021) showed that an inclusion of non-linear analysis of EEG (ApEn calculation) improved the accuracy of Neuropathic Pain assessment to 96% from 87-90% accuracy achieved with linear-only EEG analysis.

Summary observations of the evidence reviewed

The data from the studies reviewed suggest that nonlinear tools (such as ApEn and CD assessments) are already used extensively to study clinical conditions and their use has a potential to noticeably improve diagnostic accuracy (Zolezzi et. al., 2021; Adeli & Ghosh-Dastidar. 2010).

One of the results seen across the studies reviewed is a trend of increasing K-S entropy as a function of generalized arousal. Indeed, K-S entropy is increasing from deep sleep phase N3 to N2, N1, REM and wakefulness (Y. Ma et al., 2018; Lee et al., 2013; Mateos et al., 2016). K-S entropy is higher in heathy subjects than in MCS and higher in MCS than in a VS (Thul et al., 2016; Sara & Pistoia, 2010).

These results support an observation that K-S entropy can increase, decrease, or stay stable in living organisms. Specifically, K-S entropy appears to increase as generalized arousal increases from delta to gamma rhythm; K-S entropy seems to be higher in healthy, alert brain-mind functioning than it is in the states of coma, seizure, or deep sleep. K-S entropy can stay close to zero in a coma or deep sleep.

Hypothesis

The following idea seems central to the Solms (2021) theory of the mental apparatus an its functioning, and it appears to be generalized onto a wide range of living systems:

> I have conveyed three important points. The first is that the average information of a system is the entropy of that system (i.e. the entropy in a system is a measure of the amount of information needed to describe its physical state). The second is that living systems must resist entropy. These two facts together imply that we must minimise the information that we process. (Here I mean information in Shannon's sense, of course; in other words, we must minimise our uncertainty). (p.160)

The data presented in this paper suggests that the minimization of Shannon's entropy of physical states is likely not a complete picture that describes the dynamics at various scales on the brain-mind.

As one of the possible future elaborations of this idea, I propose a principle, where chaotic, stochastic, and linear processes, as well as hybrid ones, such as primarily chaotic functioning with noise can be present concurrently at different scales of the brain-mind, or at the same scale but in different places (Darbin et al., 2013; Müller et al., 2020) or at different times (Y. Ma et al., 2018). As an example, while thermoregulating homeostasis may create an upper limit on the thermodynamic entropy of the body, within this body we can have gamma rhythm in the brain, which is likely a chaotic process with some elements of noise based on the data reviewed above; K-S entropy of the scalp EEG can grow when a person progresses from deep sleep to wakefulness and then K-S entropy can remain relatively stable and positive while the person stays awake and alert.

Discussion

Mark Solms (2021) acknowledged that Shannon's entropy could increase with increased arousal:

> The 'low-arousal' patterns therefore carry less information than the 'high-arousal' ones (see Figure 10). The high-arousal ones contain more uncertainty.[21] Thus EEG entropy values are higher in minimally conscious than in vegetative patients.[22] That makes sense: cortical activity in the conscious brain communicates more information than it does during deep sleep. But here comes the strange part: if more information means more uncertainty and therefore more entropy, then – since living things must resist entropy – waking activity is less

desirable, biologically speaking, than deep sleep.23 I know this is counter-intuitive, but it will become more comprehensible as we proceed.24. (p. 158)[10]

The observation itself about waking activity being "less biologically desirable than deep sleep" is based on the premise "living things must resist entropy," which may apply to sensory entropy, but not to the entropy measures of EEG, such as SpEn or ApEn. Perhaps, this premise could be formulated more specifically and identify – which entropy exactly of which living systems, at what level of the scale, and in what circumstances?

When the brain-mind theory is not based on a generalized statement "living things must minimize entropy," there is no paradox in SpEn or ApEn entropy of EEG being higher during wakefulness as compared to sleep, since entropy variations in both directions are expected.

As mentioned earlier, homeostasis seems to be an important component in Mark Solms's "New Project" (2020a) and his approach to the "Hard Problem of Consciousness" with Karl Frison (Solms & Friston, 2018). Importantly, a homeostat at every level of the nested hierarchy would likely create an upper limit on Shannon's entropy and then the entire system consisting only of homeostats would work to minimize entropy[11].

It appears that homeostatic mechanism may be a valuable model for some processes (e.g. temperature regulation), but the data reviewed in this paper suggests that a model where a homeostat is present as a universal unit at each level of the hierarchy is likely incomplete.

Can we then consider that perception, for example, could be one of the possible processes in the brain-mind, which attempts to decrease a generalized version of Shannon's entropy of physical states (Friston et al., 2006; Friston, 2010), while other

dynamic processes, such as waking up and opening one's eyes may be accompanied by an increase in K-S entropy of scalp EEG?

To summarize, the minimization of Shannon's entropy of physical states seems to be a part of the overall picture, not a universal law that can be generalized onto all of the brain-mind functioning or even wider – to all living things. Sensory entropy minimization described by Solms and Frison (2018) might be one of the possible modes of brain-mind functioning at some level of the scale with a possibility that other modes exist.

Friston (2013) described a transition of the simulated system he created from what he called a "briefly chaotic" state onto a state converging to a global random attractor. Based on what has been presented in this paper, there is no reason why the state he described as "brief" could not be one of the stable or frequent states of the brain-mind. In other words, the system might not necessarily be always trending toward convergence – chaotic systems can be relatively stable[12] and have a divergence of trajectories in phase space.

One of the possible transitions between the primarily linear and primarily chaotic states could be happening as a function of arousal, as described in the *Hypothesis* section of this paper. As Liley et al. (2010) suggested, perhaps, this transition may occur at the level of the alpha rhythm. A possible related phenomenon that may be observed at the alpha level but also in other regimes is event related desynchronization (as reviewed in in Breakspear, 2002).

The main reason for this paper was to increase awareness of chaotic or hybrid possibilities of the brain-mind functioning, in addition to purely stochastic or purely linear ones. Based on the data reviewed, it seems that what Friston (2013) described as an "easy connection" (p.5) from the models of the brain-mind functioning that he and Solms have

developed (2018, 2020a) to the dynamical formulations including chaos theory would be indeed beneficial both theoretically and clinically[13].

Disclosure statement

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[1] As we will see later, there is an intimate relationship between chaos and non-linear dynamics.

[2] Please refer to the Glossary here:

https://static-content.springer.com/esm/art%3A10.1038%2Fs42003-019-0715-9/MediaObjects/42003_2019_715_MOESM1_ESM.pdf

[3] As you can see in Figure 2 here:

https://static-content.springer.com/esm/art%3A10.1038%2Fs42003-019-0715-9/MediaObjects/42003_2019_715_MOESM1_ESM.pdf

[4] As you can see on Figure 1 here:

https://static-content.springer.com/esm/art%3A10.1038%2Fs42003-019-0715-9/MediaObjects/42003_2019_715_MOESM1_ESM.pdf

[5] Please refer to Figure 12C on page 786 in Faure and Korn (2001), where you can see areas of linearity, stochasticity, and chaos, as well hybrid areas, such as chaos+noise.

[6] For illustrations, please refer to Figures 8 and 10 in Faure & Korn (2001).

[7] Such as disassembling a watch or a car engine to find a faulty part.

[8] Shannon wrote about a continuous version of entropy on page 35 of his paper (part III, section 20) by replacing a summation with an integral; however, it is not identical in all respects to the discrete version. E.T. Janes further elaborated continuous version of entropy as Limiting Density of Discrete Points LDDP (Janes, 1957).

[9] Two studies related to Autism Spectrum Disorder and Major Depressive Disorder diagnosis were excluded from this review due to issues with their methodology

[10] Please note that the specific entropy that Mark Solms cites in the citation [22 - Gosseries et al. (2011)] is SpEn entropy of EEG.

[11] This may be valid if each level of the scale operates in the same functional space (M. Levin, personal communications, 2022). For example, if level two is biochemical and level three is morphological then homeostats at these two levels would likely not from a nested hierarchy to collectively upper bound Shannon's entropy.

[12] Such as the Great Red Spot on Jupiter (Gleick, 2008).

[13] As one of the reviewers of this paper pointed out, this work has already started in a paper that provides a free energy principle formulation of dynamical systems that include chaos (Friston et al., 2021).